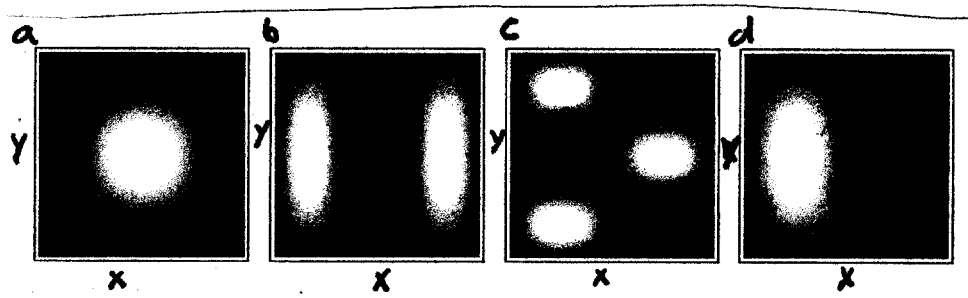
Quiz 1 | Key

① The following are density plots of the wavefunction for a particle in a 2D box. Identify these wavefunctions.



a ψ_{11} b ψ_{31} c ψ_{23} d ψ_{21}

② Calculate $\langle L_z \rangle$ for a particle on a ring. ($\psi_m = \frac{1}{\sqrt{2\pi}} e^{im\phi}$, $\hat{L}_z = -i\hbar \frac{\partial}{\partial \phi}$)

$$\langle L_z \rangle = \int_0^{2\pi} \frac{1}{\sqrt{2\pi}} e^{-im\phi} \left(-i\hbar \frac{\partial}{\partial \phi}\right) \frac{1}{\sqrt{2\pi}} e^{im\phi} d\phi = \frac{-i\hbar}{2\pi} \int_0^{2\pi} e^{-im\phi} \frac{\partial}{\partial \phi} e^{im\phi} d\phi$$

$$= \frac{m\hbar}{2\pi} \int_0^{2\pi} e^{-im\phi} e^{im\phi} d\phi = \frac{m\hbar \, 2\pi}{2\pi} = \boxed{m\hbar}$$

③ Derive an expression for the spectrum of the rigid rotor ($E_l = l(l+1)B_c$, $B_c = \frac{\hbar^2}{8\pi^2 I}$)

$$\Delta E = E_{l+1} - E_l = (l+1)(l+2)B_c - l(l+1)B_c$$

$$= (l+1)B_c[l+2-l]$$

$$= \boxed{2(l+1)B_c}$$

④ Show the superposition $\Psi = \sum a_n \psi_n$ is not an eigenfunction of $\hat{H}$ even when $\hat{H}\psi_n = E_n\psi_n$.

$$\hat{H}\Psi = \hat{H}\sum_n a_n \psi_n = \sum_n a_n \hat{H}\psi_n = \sum_n a_n E_n \psi_n \neq E\Psi$$