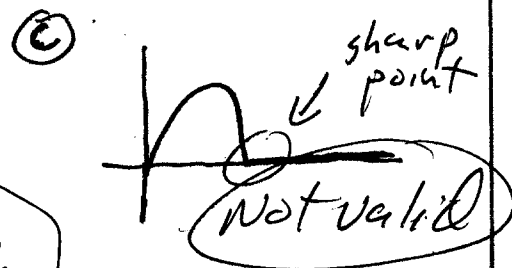
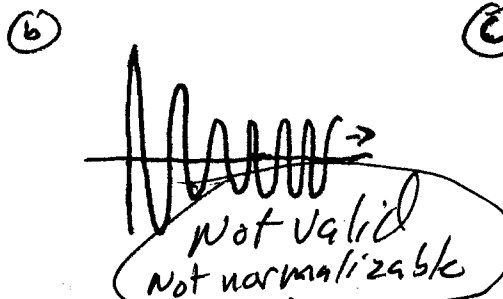
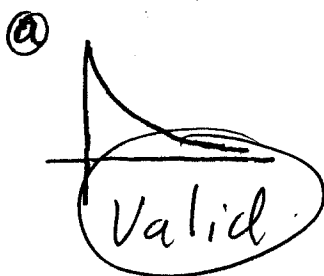


- ① State whether or not the following wavefunctions are valid. Take all space to be $0 \leq x < \infty$



- ② Normalize $\psi(x) = e^{-x}$. Take all space to be $0 \leq x < \infty$

$$N = \sqrt{\int_0^{\infty} (e^{-x})^2 dx} = \sqrt{\int_0^{\infty} e^{-2x} dx} = \sqrt{\left. \frac{1}{-2} e^{-2x} \right|_0^{\infty}}$$

$$N = \sqrt{\frac{1}{2}}$$

so

$$\psi_{\text{norm}} = \sqrt{2} e^{-x}$$

- ③ Write down the 1D Schrödinger equation for $V(x) = \frac{1}{2} kx^2$

$$\hat{H}\psi = E\psi$$

$$\hat{H} = \hat{T} + \hat{V}$$

$$\hat{T} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$$

$$\hat{V} = \frac{1}{2} kx^2$$

$$\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} kx^2 \right) \psi = E\psi$$

$$\Leftrightarrow \left(-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + \left(\frac{1}{2} kx^2 - E \right) \psi \right) = 0$$

- ④ Write out the integral that you would need to do to evaluate $\langle \hat{p}_x \rangle$ for ~~the~~ a system described by the wavefunction in problem 2 above.

$$\langle \hat{p}_x \rangle = \int_0^{\infty} \underbrace{\sqrt{2}}_{\uparrow} e^{-x} \left(-i\hbar \frac{d}{dx} \right) \underbrace{e^{-x}}_{\uparrow} dx$$

$\sqrt{2}$