$\psi_{unnorm} = N \psi_{norm}$

so $\int \psi^*_{unnorm} \psi^*_{unnorm} = N^2$

$\psi_{norm} = \frac{1}{N} \psi_{unnorm}$

Physical Chemistry
Recitation

SESSION A: Normalizing wavefunctions.

Normalize the following wavefuctions.

- $\psi(x) = \frac{\sin nx}{x}$ $\quad N^2 = \int_{-\infty}^{\infty} \psi^* \psi \, dx$

$N^2 = \int_{-\infty}^{\infty} \frac{\sin^2 nx}{x^2} dx$ From CRC Table #630 $\int_0^{\infty} \frac{\sin^2(px)}{x^2} dx = \frac{\pi p}{2}$

so $N^2 = \frac{\pi n}{\cancel{2}} \cancel{2} \Rightarrow N = \pi n$ $\quad \frac{1}{N} = \frac{1}{\sqrt{\pi n}}$ so $\boxed{\psi(x)_{norm} \frac{\sin nx}{\sqrt{\pi n}\, x}}$

- $\psi(x) = e^{-\alpha x^2}$

$N^2 = \int_{-\infty}^{\infty} e^{-2\alpha x^2} dx$ From CRC Table #663 $\int_0^{\infty} e^{-ax^2} dx = \frac{\sqrt{\pi}}{2a}$

so $N^2 = \frac{\sqrt{\pi}}{\cancel{2}(2\alpha)} \cancel{2} \Rightarrow N = \frac{\sqrt[4]{\pi}}{\sqrt{2\alpha}} \Rightarrow \frac{1}{N} = \frac{\sqrt{2\alpha}}{\sqrt[4]{\pi}}$ so $\boxed{\psi_{norm}(x) = \frac{\sqrt{\alpha}}{\sqrt[4]{\pi}} e^{-\alpha x^2}}$

- $\psi(r) = re^{-\alpha r}, \quad 0 \le r < \infty$

$N^2 = \int_0^{\infty} r^2 e^{-2\alpha r} dr$ From CRC Table #661 $\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$ n pos. int.

so $N^2 = \frac{2!}{(2\alpha)^{2+1}} \Rightarrow N = \frac{\sqrt{2}}{\sqrt{(2\alpha)^3}} \Rightarrow \frac{1}{N} = \frac{\sqrt{(2\alpha)^3}}{\sqrt{2}}$ so $\psi_{norm} = \frac{\sqrt{(2\alpha)^3}}{\sqrt{2}} r e^{-\alpha r}$

- $\psi(\theta) = e^{i\theta}, \quad 0 \le \theta < 2\pi$

$N^2 = \int_0^{2\pi} e^{-i\theta} e^{i\theta} d\theta = \int_0^{2\pi} 1 \, d\theta = 2\pi \Rightarrow N = \sqrt{2\pi} \Rightarrow \frac{1}{N} = \frac{1}{\sqrt{2\pi}}$

so $\boxed{\psi(\theta)_{norm} = \frac{1}{\sqrt{2\pi}} e^{i\theta}}$