Physical Chemistry
Recitation

$\hat{p} = -i\hbar \frac{\partial}{\partial x}$

$\hat{T} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$

SESSION B: Eigenfunctions.

Which of the following are eigenfunctions of the momentum operator (give the eigenvalues when appropriate) and which are eigenfunctions of the kinetic engergy operator (give eigenvalues when appropriate). .

- $\psi(x) = e^{ikx}$

$\hat{p}\psi(x) = -i\hbar \frac{\partial}{\partial x} e^{ikx} = \hbar k e^{ikx}$ yes, Eigenfunction → Eigenvalue = $\hbar k$

$\hat{T}\psi(x) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} e^{ikx} = \frac{\hbar^2}{2m} k^2 e^{ikx}$ yes, Eigenfunction → Eigenvalue = $\frac{\hbar^2 k^2}{2m}$

- $\psi(x) = e^{-\alpha x^2}$

$\hat{p}\psi(x) = -i\hbar \frac{\partial e^{-\alpha x^2}}{\partial x} = +i\hbar 2\alpha x e^{-\alpha x^2}$ not eigenfunction

$\hat{T}\psi(x) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} e^{-\alpha x^2} = \frac{-\hbar^2}{2m} \left[(2\alpha x)^2 e^{-\alpha x^2} + 2\alpha e^{-\alpha x^2} \right]$ not eigenfunction

- $\psi(x) = \cos kx$

$\hat{p}\psi(x) = -i\hbar \frac{\partial}{\partial x} \cos kx = +i\hbar k \sin kx$ not eigenfunction

$\hat{T}\psi(x) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \cos kx = \frac{+\hbar^2}{2m} k^2 \cos kx$ yes, eigenfunction

Eigenvalue = $+\frac{\hbar^2 k^2}{2m}$

- $\psi(x) = kx$

$\hat{p}\psi(x) = -i\hbar \frac{\partial}{\partial x} kx = -i\hbar k$ not eigenvalue

$\hat{T}\psi(x) = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} kx = 0$ This is tough.

yes eigenfunction

eigenvalue = 0