1. Find $\frac{df}{dx}$ for the following by using the chain rule:

   (a) $f = (1-x)^3$ $\quad \frac{df}{dx} = -3(1-x)^2$

   (b) $f = (x^2-4)^6$ $\quad \frac{df}{dx} = 12x(x^2-4)^5$

   (c) $f = \cos^4 x$ $\quad \frac{df}{dx} = -4\sin x \cos^3 x$

   (d) $f = \sin^2 x \cos x$ $\quad \frac{df}{dx} = 2\sin x \cos^2 x - \sin^3 x$

2. Differentiate $y = 1/(x^2-2)$ first by the quotient rule and then by writing $y = 1/(x^2-2) = (x^2-2)^{-1}$ and using the chain rule. $\frac{dy}{dx} = \frac{-2x}{(x^2-2)^2}$

3. Prove the quotient rule by applying the chain rule to $\frac{f}{g} = fg^{-1}$.

4. Find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ via *implicit differentiation* for the following equations:

   (a) $xy - y - x = 0$ $\quad \frac{dy}{dx} = \frac{1-y}{x-1} \quad \frac{dx}{dy} = \frac{1-x}{y-1}$

   (b) $2x^2 - 2xy + y^2 = 5$ $\quad \frac{dy}{dx} = \frac{y-2x}{y-x} \quad \frac{dx}{dy} = \frac{x-y}{2x-y}$

   (c) $y - x\sin y + 1 = \tan x$ $\quad \frac{dy}{dx} = \frac{\sec^2 x + \sin y}{1 - x\cos y} \quad \frac{dx}{dy} = \frac{1 - x\cos y}{\sec^2 x + \sin y}$

5. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and df (the total differential) for the following:

   (a) $f(x,y) = xy - y - x$ $\quad \frac{\partial f}{\partial x} = y-1, \ \frac{\partial f}{\partial y} = x-1 \quad df = (y-1)dx + (x-1)dy$

   (b) $f(x,y) = 2x^2 - 2xy + y^2$ $\quad \frac{\partial f}{\partial x} = 4x - 2y, \ \frac{\partial f}{\partial y} = -2x + 2y \quad df = (4x-2y)dx + 2(x-y)\,dy$

   (c) $f(x,y) = y - x\sin y + 1$ $\quad \frac{\partial f}{\partial x} = -\sin y, \ \frac{\partial f}{\partial y} = 1 - x\cos y \quad df = -\sin y\, dx + (1 - x\cos y)\, dy$

6. Find and classify the extrema for the follow functions:

   (a) $f(x) = x^2 + 2x$ $\quad x = -1 \quad$ min

   (b) $f(x) = x^3$ $\quad x = 0 \quad$ inflection

   (c) $f(x) = x$ $\quad$ no extrema

   (d) $f(x,y) = x^2 - y^2$ $\quad (0,0) \quad$ saddle

7. Expand the following functions in a Taylor series about the origin. Make sure to include the *radius of convergence*.

   (a) $f(x) = 1/(1+x) = 1 - x + x^2 - x^3 \ldots$ $\quad -1 < x \leq 1$

   (b) $f(x) = \sin x$ $\quad = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$ $\quad x \leq \infty$

   (c) $f(x) = \cos x$ $\quad = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$ $\quad x \leq \infty$

8. Integrate the following functions from $x = a$ to $x = b$.

   (a) $f(x) = x^2 + c$ $\quad \int_a^b f(x) = \frac{1}{3}(b^3 - a^3) + c(b-a)$

   (b) $f(x) = \sin x$ $\quad \int_a^b f(x) = \cos a - \cos b$

   (c) $f(x) = \cos x$ $\quad \int_a^b f(x) = \sin b - \sin a$

9. Integrate the Taylor series of $\cos x$ and $\sin x$. What functions do the resulting series represent.