Practice Quiz | Key

① A six-sided loaded die has a weight function $w(n) = \frac{1}{n}$. How much more likely are you to roll an odd number?

$$P_1 = \frac{1}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}} \qquad P_2 = \frac{\frac{1}{2}}{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}} \quad \text{etc.}$$

$P_1 = 0.408$ $P_2 = 0.204$ $P_3 = 0.136$ $P_4 = 0.102$

$P_5 = 0.082$ $P_6 = 0.068$

$P_{odd} = P_1 + P_3 + P_5 = 0.626$

$P_{even} = P_2 + P_4 + P_6 = 0.374$

1.67 times more likely to roll an odd number

② The C-Cl stretching in carbon tetrachloride (CCl_4) is about 470 cm^{-1}. What is the ratio of the first excited state population to the ground state population?

$$\frac{P_1}{P_0} = e^{-\hbar\omega\beta} = e^{\frac{-470\,cm^{-1}}{207\,cm^{-1}}} = 0.103$$

③ It was determined that the ratio of the first excited state population to the ground state population for the "ring breathing" mode of benzene (992 cm^{-1}) was 2.90×10^{-2}. What is the temperature of the sample?

$$\frac{P_1}{P_0} = 2.90 \times 10^{-2} = e^{\frac{-992\,cm^{-1}}{kT}} \xrightarrow{\text{ln both sides}} -3.54 = -\frac{992}{kT}$$

$$kT = 280\ cm^{-1} \Rightarrow T = \frac{(280\,cm^{-1})\left(3.0 \times 10^{10}\,\frac{cm}{s}\right)(6.63 \times 10^{-34}\,J\,s)}{1.38 \times 10^{-23}\,\frac{J}{K}}$$

$T = 404\ K$