Problem Set PS02

ISSUED: 9/10/09 **Due: 9/17/09**

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Instructions. Complete all questions before class on the due date. You are encouraged to work together. Be sure to struggle with the problem before seeking help. Many of the exercises are very similar to problems in the book. Understanding the solution to these problems will be helpful in completing the assigned exercises.

Mathematical Exercises

1. The zeros of functions are important points. The zeros of wavefunctions are called *nodes* which you have probably heard about in freshman chemistry.

   (a) Find the value of a such that $f(x) = x^2 - a$ has zeros at $x = \pm 3$.

   (b) Find the values of a_n such that $f(x) = x \cos a_n x$ has zeros at $x = 0$ and $x = L$.

2. The functions

$$y_1 = Ae^{ikx}$$

   and

$$y_2 = Be^{-ikx}$$

   are solutions of the differential equation

$$\frac{d^2y}{dx^2} + k^2 y = 0, \tag{1}$$

   where k is a positive real constant. Verify by direct substitution that any *linear combination* of the independent solutions of Eq. (1) thus, $Y = y_1 + y_2$, is a solution of Eq. (1). But show that the product $Z = y_1 y_2$ is not a solution. Also show that

$$y_3 = Ce^{kx}$$

   is not a solution.

3. Suppose the following *boundary conditions* are imposed on the solutions of Eq. (1), $Y(0) = 1$ and $Y(\pm L/2) = 0$. What are A, B and k.

4. Suppose some crazy family of operators, $\left\{\hat{M}_n\right\}$ is defined such that $\hat{M}_n f(x)$ means take the n^{th} moment of $f(x)$. Write out the appropriate integral for the action of $\hat{M}_n$ on the following functions:

   (a) $\psi(x) = 1/x^2$, all space $1 < x < 10$.

   (b) $\psi(x) = kx^2$, all space $-1 < x < 1$.

   (c) $\psi(x) = e^{-x}$, all space $0 < x < \infty$.

Exercises

5. Suppose that the wavefunction for an electron of some crazy system over the region $(0 < x < \infty)$ is of the form $\psi(x) = (x^2 - 1)e^{-x}$. You can use MATHEMATICA to help with the integrals.

   (a) Normalize this wavefunction over this region.

   (b) Find the probability that the electron is more than a distance $x = 1$ away from $x = 0$.

   (c) Find the probability that the electron is between $x = 1$ and $x = 2$.

6. Often one is interested in extracting the average value of some property from a wavefunction. This is done using the average value theorem which states that the average value of some observable, $\hat{O}$, is given by

$$\left\langle \hat{O} \right\rangle = \int_{\substack{\text{all}\\\text{space}}} \psi^*_{\text{norm}} \hat{O} \psi_{\text{norm}}, \tag{1}$$

   where the angled brackets denote averaging. Considering the wavefunction given above, find the average value of $\hat{x}$ and $\hat{x}^2$. Be careful, what is all space for this problem.

7. For the above averages it was not important to worry about putting the opperator in between the wavefunctions. For the operator $d = \frac{d}{dx}$, it is important because the derivative needs to be taken on only the wavefunction to the right of the opperator. Evaluate $\left\langle \hat{d} \right\rangle$ for the wavefunction of the previous problems.

8. The uncertainty of the average value of an observable is denoted as δO and is given by

$$\delta O = \sqrt{\left\langle \hat{O}^2 \right\rangle - \left\langle \hat{O} \right\rangle^2}. \tag{2}$$

   Find δx for the wavefunction used in the previous problems.

Conceptual Problems

9. Use the uncertainty principle to explain why atoms are stable, i.e., do not collapse.

10. Consider the family of wavefunctions $\psi(x) = x^n e^{-x}$, $n = 0, 1, 2 \ldots$ over the range $(0 < x < \infty)$. Without calculating any integrals, how does the probability of finding the particle between 0 and 1 go with n. It is very helpful to plot several of these functions to get an idea of how they behave with n.

11. Why must one normalize wavefunctions.

Computer Problems

12. Use MATHEMATICA to plot the family of functions $f(x) = \sin n\pi x$, where $n = 1, 2, 3 \cdots$ from 0 to 1. Do all your plots share $x = 0$ and $x = 1$ as common zeros? Plot the first five functions.

13. Similar to the previous problem, use MATHEMATICA to plot a family of sin functions that share $x = 0$ and $x = 3/4$ as common zeros. Limit your plot range from 0 to 3/4

14. A quantum object on the range $-\infty < x < \infty$ is described by the set of wavefunctions $\psi_n(x) = x^n e^{-x^2}$, where $n = 1, 2, 3 \cdots$. Use MATHEMATICA to normalize these $\psi_n(x)$ for the cases of $n = 1, 2$ and 3. Then plot $\psi_n(x)$ and $|\psi_n(x)|^2$ for the cases of $n = 1, 2$ and 3.

Application of Physical Chemistry to Neuroscience

15. Read sections 1.3–1.5 of the neuroscience workbook.

16. Do problems 1.9–1.12 in the neuroscience workbook.

Reflective Exercises

17. During a casual conversation, a colleague from another department stated emphatically that science can not explain love. I replied 'not yet anyway.' If, down the road, science can completely describe the chemical and physical reactions that elicit love and can, in fact, reproducibly produce love in all its various forms, will that make it any less beautiful or any less useful to society.

18. Please read the following dictum: 'As one goldfish said to another: "Of course there is a God! Who do you think changes the water?"'

    (a) To what extent is the goldfish's conception of God a good one.

    (b) In principle, goldfish scientists could, through careful experiment and observation, eventually determine that the waterchanger is simply a human. Would that knowledge in any way change the answer to (a)?