

Problem Set PS01
ISSUED: 9/3/09 Due: 9/10/09

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Name _____

Instructions. Complete all questions before class on the due date. You are encouraged to work together. Be sure to struggle with the problem before seeking help. Many of the exercises are very similar to problems in the book. Understanding the solution to these problems will be helpful in completing the assigned exercises.

Mathematical Exercises

1. Find $\frac{df(x)}{dx}$ for the following functions. You will need to use the chain rule
 - (a) $f(x) = e^{-x} \cos x^2$
 - (b) $f(x) = x^2 e^{e^x}$
2. The n^{th} moment of a function is defined as $\int_{\text{all space}} x^n f(x) dx$. Determine the first and second moments of $f(x) = e^{-ax^2}$ where all space is $-\infty < x < \infty$. You can use MATHEMATICA to help you with the integral if you would like. This function is a very important one; it is called a Gaussian and is ubiquitous in all areas of science.
3. If $f(x)$ in the previous problem is a probability distribution, the moments give statistical information about that distribution. The first moment gives the average value of x . The second moment describes the width of the distribution about the average. Roughly speaking 2 times the square root of this value is the full width at half height of the distribution. The third moment gives the skewness of the distribution with a negative value skewing to the left and a positive value skewing to the right. Finally, the fourth moment gives the kurtosis which is the flatness of the peak relative to a Gaussian distribution. Plot the function

$$f(x) = e^{-x^4 - x^3 - x^2 - x - \frac{2}{3}}.$$

Then numerically integrate the first four moments using the code (example shows the fourth moment, adjust accordingly),

```
NIntegrate[x^4 Exp[-x^4 - x^3 - x^2 - x - 2/3], {x, -15, 15}]
```

Comparing the values for the moments to your plot, do the numbers make sense?

4. Find the zeros of the following functions. (The zeros are the values of x where $f(x) = 0$). Also plot these functions.
 - (a) $f(x) = x^2 - 9$
 - (b) $f(x) = x^2 e^{-ax^2}$, $a > 0$.
 - (c) $f(x) = x^3 - x^2 - x$.

5. Sketch a plot of the following functions and their derivatives. Is the function continuous? Is the derivative continuous?

(a) $f(x) = \Theta(x)e^{-x}$ where $\Theta(x)$ is the step function defined as

$$\Theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases}$$

(b) $f(x) = e^{-|x|}$

(c) $f(x) = e^{-x^2}$

(d) $f(x) = \frac{\sin x}{x}$, note this function is called the sinc (pronounce 'sink') function.

6. One can write $e^{ix} = \cos x + i \sin x$.

(a) Use this fact to derive the Euler identities $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$ and $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$. Hint: you will need to think about the symmetry of cosine and sine. That is, what is $\cos(-x)$ and $\sin(-x)$?

(b) The hyperbolic functions $\cosh x$ and $\sinh x$ are related to the trig functions as

$$\cosh x = \cos ix$$

and

$$\sinh x = -i \sin ix.$$

Use the Euler identities and these relations to derive Euler-like identities for $\cosh x$ and $\sinh x$.

7. What is the real part and imaginary part of the following functions

(a) $e^{3ix} \sin 3x$

(b) $x^2 + 6ix + 3$

(c) $\frac{1}{x+i\gamma}$, γ is a real constant.

Conceptual Problems

8. Assume that we performed a four slit experiment. Qualitatively sketch the outcome of the following experiments

(a) We make no attempt to determine which slit the electron went through.

(b) We block the middle two slits.

(c) We are able to determine that the electron went through one of the right two slits or one of the left two slits, but of the two, we don't know which one.

(d) We setup devices on two of the slits to monitor electrons passing through them.

9. Explain in your own words how the six experiments listed on page 2–3 of the notes posed a problem to turn of the century physics and briefly how quantum mechanics explains these phenomena.

10. State whether or not the following functions are valid wavefunctions over the range specified. If you say not valid explain why.
- (a) $\psi(x) = xe^{-x^2}$, $-\infty \leq x \leq \infty$
 - (b) $\psi(x) = \frac{\sin x}{x}$, $-\infty \leq x \leq \infty$, be careful. Note, $\frac{\sin x}{x}$ has a special name; it is called $\text{sinc } x$.
 - (c) $\psi(x) = \frac{\cos x}{x}$, $-\infty \leq x < \infty$, be careful. Note, $\frac{\cos x}{x}$ is NOT call $\text{cosec } x$.
 - (d) $\psi(x) = x^{1/2}$, $0 \leq x < 1$, Hint: invert the equation $y = x^{1/2}$ i.e., solve for x .
 - (e) $\psi(x) = xe^{-x^2+ix}$, $-\infty \leq x < \infty$, Hint: recall Eulers identity. Also, both the real part and imaginary part must be valid.

Computer Problems

11. Plot the wavefunctions of problem 10. Some of these wavefunctions are complex, so you will need to separately plot the real and imaginary parts. Use the commands `Re []` and `Im []` for this
12. Use MATHEMATICA to help you normalize the following wavefunctions over all space ($-\infty \leq x \leq \infty$). Plot the probability distribution associated with each of these wavefunctions. Qualitatively give the most probable position and average position for each of the distributions.
- (a) $\psi(x) = xe^{-x^2}$, note: your result should involve a function called the gamma function (denoted as Γ).
 - (b) $\psi(x) = x^2e^{-x^4}$
 - (c) $\psi(x) = \frac{\sin x}{x}$.

Application of Physical Chemistry to Neuroscience

13. Read sections 1.1 and 1.2 of the neuroscience workbook.
14. Do problems 1.1–1.8 in the neuroscience workbook.

Reflective Exercises

15. The current definitions of science as set forth by the American Physical Society Panel on Public Affairs (APS–POPA) are copied below.
- (a) Give one example of how quality science extends and enriches our lives. Also give an example where quality science has failed to do this.
 - (b) Give one example of how quality science expands our imagination.
 - (c) Give one example of how quality science liberates one from the bonds of ignorance and superstition.
 - (d) Do the above definitions jive with your definition(s) of science?

“Science extends and enriches our lives, expands our imagination and liberates us from the bonds of ignorance and superstition. The American Physical Society affirms the precepts of modern science that are responsible for its success.

“Science is the systematic enterprise of gathering knowledge into testable laws and theories.

“The success and credibility of science are anchored in the willingness of the scientist to:

i. Expose their ideas and results to independent testing and replication by other scientists. This requires the complete and open exchange of data, procedures and materials.

ii. Abandon or modify accepted conclusions when confronted with more complete or reliable or observational evidence.

Adherence to these principles provides a mechanism for self correction that is the foundation of the credibility of science”

16. There are many hotbutton issues in scientific research that extend into the public debate. Clonning and stem cell research are among the most controversial.
 - (a) Many people involved in these debates are outside of the scientific community. Should scientific progress be at all influenced by non-scientists. List advantages and disadvantages of this outside influence. List advantages and disadvantages of allowing only scientists to determine the progress of science.
 - (b) Do you think moral and social values present in the scientific community are representative of larger society. If not, how do you suspect they deviate.