

Problem Set PS09
Issued: 11/7/02 Due: 11/14/02

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Name _____

Instructions. Complete all questions before class on the due date. You are encouraged to work together. Be sure to struggle with the problem before seeking help. Many of the exercises are very similar to problems in the book. Understanding the solution to these problems will be helpful in completing the assigned exercises.

Exercises

1. One mole of an ideal gas in a piston expands irreversibly from 1.75L to 4.25L under a constant pressure of 1atm. (The molar heat capacity of the gas is $C_m = \frac{5}{2}R$, where R is the gas constant.)
 - (a) How much work was done?
 - (b) If the piston has adiabatic walls, what is the change in temperature of the gas?
 - (c) If the piston has diathermic walls, how much heat is exchanged between the piston and its environment?
2. One mole of an ideal gas in a piston expands reversibly and isothermally from 1.75L to 4.25L. (The molar heat capacity of the gas is $C_m = \frac{5}{2}R$, where R is the gas constant.) How much work was done?
3. Starting with the first law, derive an expression for heat taken in during reversible, isothermal expansion of an ideal gas.
4. Consider reversible adiabatic expansion of an ideal gas

(a) Show

$$dU = -\frac{nRT}{V}dV.$$

(b) Unfortunately, for adiabatic expansion, T is not a constant but we can make some progress by recognizing $dU = CdT$. Take heat capacity to be constant and the first conceptual problem to obtain the approximate expression

$$C \ln \frac{T_2}{T_1} = -nR \ln \frac{V_2}{V_1}.$$

5. An ideal gas in a piston is reversibly heated at a pressure of 1.0atm. If the piston initially had a volume of 1.0L and a temperature of 200K, what is the final volume and temperature if 30J of heat energy is added? How much work was done? (The heat capacity of the gas is $C = \frac{5}{2}R$, where R is the gas constant.)
6. Derive the equation of state for internal energy based on a Berthelot gas. (hint: see pages 110 and 111 of the notes)

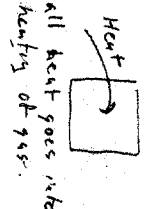
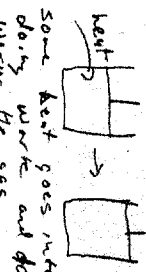
Conceptual Problems

7. Convince yourself that the internal energy of an ideal gas is independent of volume.
8. A smart and motivated freshman is interested in learning more about heat capacity. Explain to the freshman why it is important to consider the difference between heat capacity at constant pressure and heat capacity at constant volume.
9. Why is the C_P/C_V greater for a gas than a liquid or solid?

Reflective Questions

10. Visit <http://www.phds.org/ratings/> and select the subject that is of the most interest to you (if none interest you choose chemistry) then click on “next.” Select the weighting criteria that are most important to you then click on “next.” What are the top ten schools in your field?
11. As you leave Concordia, your academic or professional life will focus considerably. This has advantages and disadvantages. The focussing of effort greatly increases your ability to learn your profession. Unfortunately focussing can erode some of the breadth of interest that you have here. Think of some specific ways that you might maintain your broad interests without hindering your progression in your field.
12. Think about how you prioritize the following currently and how your priorities might change as you progress through your professional and personal life. Think about external pressures that will demand a change in your priorities and how you might resolve such conflicts.
 - work
 - family
 - wealth
 - personal happiness
 - faith/spirituality
 - service to society
 - fame
 - success
 - personal health
 - personal freedom

- ① $w = -P_{ext} \Delta V = -1 \text{ atm} (2.5 \text{ L}) = \boxed{-2.5 \text{ L}\cdot\text{atm}}$
- ② $\Delta U = w = -2.5 \text{ L}\cdot\text{atm}$ $\frac{\Delta U}{T} = n C_{V,m}$
 $\Delta T = \frac{\Delta U}{n C_{V,m}} = \frac{-2.5 \text{ L}\cdot\text{atm}}{1 \text{ mol} (\frac{5}{2}) (\frac{0.082 \text{ L}\cdot\text{atm}}{\text{K}\cdot\text{mol}})} = \boxed{-12.2 \text{ K}}$
- ③ $\Delta U = w + q$ $\Delta U = 0$ so $q = -w = \boxed{2.5 \text{ L}\cdot\text{atm}}$
- ④ $w = -nRT \ln \frac{V_2}{V_1} = -1 \text{ mol} (0.082 \frac{\text{L}\cdot\text{atm}}{\text{K}\cdot\text{mol}}) (300 \text{ K}) \ln \frac{4.25}{1.75} = \boxed{-131.8 \text{ L}\cdot\text{atm}}$
- ⑤ $\Delta U = dq + dw$ $nV, \text{ isothermal} \Rightarrow \Delta U = dq - PdV$
 $dq = PdV$ $dq = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{nRT}{V} dV$
 $\boxed{dq = nRT \ln \frac{V_2}{V_1}}$
- ⑥ $\Delta U = dq + dw$ $adiabatic$ $dU = dw \Rightarrow dU = -PdV$
 $dU = -\frac{nRT}{V} dV$ n an ideal gas law ✓
- ⑦ $dU = -\frac{nRT}{V} dV \Rightarrow \int \frac{C_V dT}{T} = -\frac{nR}{V} dV$
 $\int_{T_1}^{T_2} \frac{C_V dT}{T} = \int_{V_1}^{V_2} -\frac{nR}{V} dV$ $\boxed{C_V \frac{T_2}{T_1} = -nR \ln \frac{V_2}{V_1}}$
- ⑧ $PV = nRT$ $n = \frac{PV}{RT} = \frac{(1 \text{ atm})(1 \text{ L})}{(0.082 \frac{\text{L}\cdot\text{atm}}{\text{K}\cdot\text{mol}})(300 \text{ K})} = 6.1 \times 10^{-2} \text{ mol}$
 Need $C_{p,m}$ not $C_{V,m}$ $C_{p,m} = C_{V,m} + R$ for ideal gas
 So $\Delta T = \frac{q}{n C_{p,m}} = \frac{30 \text{ J}}{(6.1 \times 10^{-2} \text{ mol})(8.314 \frac{\text{J}}{\text{K}\cdot\text{mol}})} = 16.9 \text{ K}$
 $\Delta V = \frac{nR \Delta T}{P} = 8.45 \times 10^{-2} \text{ L}$ so $\boxed{V_2 = 1.085 \text{ L}}$
 $w = -P_{ext} \Delta V = -8.45 \times 10^{-2} \text{ L}\cdot\text{atm} = \boxed{4.57 \text{ J}}$

- ⑨ $dU = C_V dT + (P - Pd) dV$ $P = \frac{0.082}{V - n b} - \frac{n^2 a}{V^2}$
 $dU = C_V dT + \left[\frac{nR}{V - n b} + \frac{n^2 a}{V^2 T^2} \right] - \frac{nRT}{V - n b} - \frac{n^2 a}{V^2 T} dV$
 $\boxed{dU = C_V dT + \frac{n^2 a}{V^2 T} dV}$
- ⑩ As far as an ideal gas particle is concerned, it is the only particle in the container. Also all its energy is kinetic and it undergoes elastic collisions with the wall. Changing the size of the container will not affect its kinetic energy if the particle if the temperature is held constant so V should be independent of V .
- ⑪ $\text{Coast } + V$ $\text{Coast } P$
- ⑫ 
 all heat goes into heating of gas.
- ⑬ 
 Some heat goes into doing work and does not warm the gas.
- ⑭ A gas expands with T much more than does a liquid or solid. This expansion does work.
- ⑮ - ⑯ your words