Problem Set PS01

Issued: 8/29/02 **Due: 9/5/02**

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Instructions. Complete all questions before class on the due date. You are encouraged to work together. Be sure to struggle with the problem before seeking help. Many of the exercises are very similar to problems in the book. Understanding the solution to these problems will be helpful in completing the assigned exercises.

Mathematical Exercises

1. Find $\frac{df(x)}{dx}$ for the following functions

   (a) $f(x) = x^2 e^{x^2}$

   (b) $f(x) = e^{a \cos x}$

   (c) $f(x) = g(h(j(x)))$

2. Find $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$ for the following functions

   (a) $f(x,y) = \frac{x^2 - y^2}{x+y}$

   (b) $f(x,y) = xye^{y^2}$

   (c) $f(x,y,u,v) = \frac{2x - xy}{u - v}$

3. Find the zeros of the following functions. (The zeros are the values of x where $f(x) = 0$). Also plot these functions.

   (a) $f(x) = x^2 - 36$

   (b) $f(x) = e^{-x^2} \sin x$

   (c) $f(x) = \sin \frac{x}{k}$, k is a constant.

4. One can write $e^{ix} = \cos x + i \sin x$. Use this fact to derive the Euler identities $\sin x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right)$ and $\cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$. Hint: you will need to think about the symmetry of cosine and sine. That is, what is $\cos(-x)$ and $\sin(-x)$? The hyperbolic functions $\cosh x$ and $\sinh x$ are related to the trig functions as

   $$\cosh x = \cos ix$$

   and

   $$\sinh x = -i \sin ix.$$

   Use the Euler identies and these relations to derive Euler–like identities for $\cosh x$ and $\sinh x$.

5. A complex number z can be written as $z = x + iy$ where x is the real part of z ($x = \text{Re}[z]$) and y is the imaginary part of z ($y = \text{Im}[z]$). A complex number can also be represented by a point in the complex plane. The complex plane is defined by a set of Cartesian coordinates (x, y) such that the point (x_1, y_1) in the complex plane corresponds to the complex number $z = x + iy$. Graph the complex numbers 0.5, $2 - i$, $0.5i$, $-2 + i$, $\sqrt{-4}$, e^{0i}, $e^{\frac{\pi i}{2}}$, $e^{\pi i}$, $e^{\frac{3\pi i}{2}}$, $e^{2\pi i}$ in the complex plane. Hint: It may be helpful for the last five points if you express the exponentials as sines and cosines like the previous problem.

6. What is the real part, imaginary part, and amplitude (or modulus) of the following functions

   (a) $e^{2ix} \cos x$
   (b) $x^2 + 3ix + 2$
   (c) $\frac{1}{x+i\gamma}$, γ is a real constant.

Exercises

7. Sketch a plot of the Paschen series spectrum for the hydrogen atom.

8. The Bohr model works well for *hydrogenic systems* (a nucleus and one electron). Sketch the "Paschen series" for He^{+} and Li^{2+}. Also, what is the ionization energy for these ions?

Conceptual Problems

9. Explain in your own words how the six experiments listed on page 2–3 of the notes posed a problem to turn of the century physics and briefly how quantum mechanics explains these phenomena.

10. Assume that we performed a three slit experiment. Qualitatively sketch the outcome of the following experiments

    (a) We make no attempt to determine which slit the electron went through.
    (b) We block the middle slit.
    (c) We block two slits.
    (d) We setup devices on two of the slits to monitor electrons passing through them.

11. State whether or not the following functions are valid wavefunctions over the range specified. If you say not valid explain why.

    (a) $\psi(x) = ie^{-x^2}$, $-\infty \leq x \leq \infty$
    (b) $\psi(x) = \frac{\sin x}{x}$, $-\infty \leq x \leq \infty$, be careful. Note, $\frac{\sin x}{x}$ has a special name; it is called $\text{sinc}\, x$.
    (c) $\psi(x) = \frac{\cos x}{x}$, $-\infty \leq x < \infty$, be careful. Note, $\frac{\cos x}{x}$ is NOT call $\text{cosc}\, x$.
    (d) $\psi(x) = \sqrt{x}$, $0 \leq x < 1$
    (e) $\psi(x) = e^{-x^2+ix}$, $-\infty \leq x < \infty$

Computer Problems

12. Check Mathematical Exercises 1–3 using MATHEMATICA.

13. Use MATHEMATICA to help you normalize the following wavefunctions over all space ($-\infty \leq x \leq \infty$). Plot the probability distribution associated with each of these wavefunctions. Qualitatively give the most probable position and average position for each of the distributions.

    (a) $\psi(x) = xe^{-x^4}$, note: your result should involve a function called the gamma function (denoted as Γ).

    (b) $\psi(x) = x^6 e^{-x^2}$

    (c) $\psi(x) = e^{-x^2} J_0(10x)$, where J_0 is called the zeroth order Bessel function of the first kind. Note: your result should involve a generalized hypergeometric function (denoted as ${}_pF_q$).

Reflective Exercises

14. The current definitions of science as set forth by the American Physical Society Panel on Public Affairs (APS–POPA) read as follows:

    "Science extends and enriches our lives, expands our imagination and liberates us from the bonds of ignorance and superstition. The American Physical Society affirms the precepts of modern science that are responsible for its success.
    "Science is the systematic enterprise of gathering knowledge into testable laws and theories.
    "The success and credibility of science are anchored in the willingness of the scientist to:
    i. Expose their ideas and results to independent testing and replication by other scientists. This requires the complete and open exchange of data, procedures and materials.
    ii. Abandon or modify accepted conclusions when confronted with more complete or reliable or observational evidence.
    Adherence to these principles provides a mechanism for self correction that is the foundation of the credibility of science"

    (a) Give one example of how quality science extends and enriches our lives. Also give an example where quality science has failed to do this.

    (b) Give one example of how quality science expands our imagination.

    (c) Give one example of how quality science liberates one from the bonds of ignorance and superstition.

    (d) Do the above definitions jive with your definition(s) of science?

15. In mid-August of last year, President G.W. Bush announced that the US government would not support embryonic stem cell research (except for the currently existing embryonic stem cell lines). Along with the possibility of human cloning, embryonic stem cells have been at the center of heated debate over the moral and social consequences of modern science.

    (a) If you do not know about this embryonic stem cell debate learn more about it.

    (b) Many people involved in this debate are outside of the scientific community. Should scientific progress be at all influenced by non-scientists. List advantages and disadvantages of this outside influence. List advantages and disadvantages of allowing only scientists to determine the progress of science.

    (c) Do you think moral and social values present in the scientific community are representative of larger society. If not, how do you suspect they deviate.

① ⓐ $\frac{d\, x^2 e^{x^2}}{dx} = x^2 \frac{d e^{x^2}}{dx} + \left(\frac{dx^2}{dx}\right) e^{x^2}$

$= x^2 (2x e^{x^2}) + 2x e^{x^2} = (2x^3 + 2x) e^{x^2}$

ⓑ $\frac{d e^{a \cos x}}{dx} = -a \sin x \, e^{a \cos x}$

ⓒ $\frac{dg(h(j(x)))}{dx} = \frac{dj}{dx} \frac{dh}{dj} \frac{dg}{dh}$

② ⓐ $\frac{\partial}{\partial x}\left(\frac{x^2-y^2}{x+y}\right) = \frac{(x^2-y^2) - 2x(x+y)}{(x+y)^2}$

$\frac{\partial}{\partial y}\left(\frac{x^2-y^2}{x+y}\right) = \frac{(x^2-y^2) + 2y(x+y)}{(x+y)^2}$

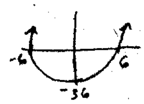
ⓑ $\frac{\partial x y e^{y^2}}{\partial x} = y e^{y^2}$ $\quad \frac{\partial}{\partial y} x y e^{y^2} = xy(2y) e^{y^2} + x e^{y^2}$

$= (2y^2+1) x e^{y^2}$

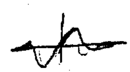
ⓒ $\frac{\partial}{\partial x}\left(\frac{2x - xy}{u - v}\right) = \frac{2 - y}{u - v}$

$\frac{\partial}{\partial y}\left(\frac{2x - xy}{u - v}\right) = \frac{-x}{u - v}$

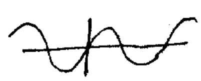
③ ⓐ $x^2 - 36 = 0 \Rightarrow x = \pm 6$



ⓑ $e^{-x^2} \sin x$ $\quad e^{-x^2} > 0 \;\forall x$ so $\sin x = 0$ at $x = n\pi$, $n = 0, \pm1, \pm2 \ldots$



ⓒ $\sin \frac{x}{k} = 0 \Rightarrow x = nk\pi \quad n = 0, \pm1, \pm2 \ldots$



④ $e^{ix} = \cos x + i \sin x \qquad e^{-ix} = \cos -x + i \sin -x = \cos x - i \sin x$

$\frac{1}{2}(e^{ix} + e^{-ix}) = \frac{1}{2}(\cos x + i \sin x + \cos x - i \sin x) = \cos x$ ✓

$\frac{1}{2i}(e^{ix} - e^{-ix}) = \frac{1}{2i}(\cos x + i \sin x - \cos x + i \sin x) = \sin x$ ✓

$\cos ix = \frac{1}{2}(e^{i(ix)} + e^{-i(ix)}) = \boxed{\frac{1}{2}(e^{-x} + e^{x}) = \cosh x}$

$-i \sin ix = -\frac{1}{2i}(e^{i(ix)} - e^{-i(ix)}) = \boxed{-\frac{1}{2}(e^{-x} - e^{x}) = \sinh x}$

⑤ see last year's solution key

⑥ ⓐ $e^{2ix} \cos x = (\cos 2x + i \sin 2x) \cos x$

Real part = $\cos x \cos 2x$

Im part = $\sin 2x \cos x$ $\quad$ Note: Not $i \sin 2x \cos x$ *

mod = $\sqrt{e^{2ix} \cos x \, e^{-2ix} \cos x} = \cos x$

ⓑ real part = $x^2 + z$

Im part = $3x$

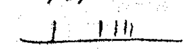
mod = $\sqrt{(x^2+z)^2 + (3x)^2}$

ⓒ real part = $\frac{1}{2}\left(\frac{1}{x+iy} + \frac{1}{x-iy}\right) = \frac{x}{x^2+y^2}$

Im part = $\frac{1}{2i}\left(\frac{1}{x+iy} - \frac{1}{x-iy}\right) = \frac{-y}{x^2+y^2}$

mod = $\sqrt{\left(\frac{1}{x+iy}\right)\left(\frac{1}{x-iy}\right)} = \sqrt{\frac{1}{x^2+y^2}}$

⑦ Paschen $\Delta E = R\left(\frac{1}{n^2} - \frac{1}{3^2}\right) \quad n = 4, 5, 6 \ldots$



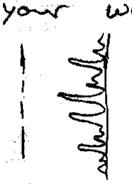
⑧ $\Delta E_{He^+} = -4R\left(\frac{1}{n^2} - \frac{1}{3^2}\right)$

$\Delta E_{Li^{2+}} = -9R\left(\frac{1}{n^2} - \frac{1}{3^2}\right)$

He⁺ Dashed Li^{2+} solid

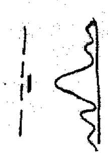
⑨ your words

⑩ ⓐ



Full interference of 3 waves

ⓑ

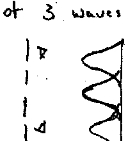


interference of 2 waves

ⓒ

No interference

ⓓ



No interference

⑪ ⓐ Valid ⓑ valid (no problem at x=0)

ⓒ Not valid blows up at x=0

ⓓ Valid ⓔ Valid

⑫

In[86]:= D[x^2 Exp[x^2], x]

Out[86]= $2 e^{x^2} x + 2 e^{x^2} x^3$

In[87]:= D[Exp[a Cos[x]], x]

Out[87]= $-a\, e^{a \cos[x]} \sin[x]$

In[88]:= D[g[h[j[x]]], x]

Out[88]= g'[h[j[x]]] h'[j[x]] j'[x]

In[89]:= D[(x^2 - y^2) / (x + y), x]

Out[89]= $\frac{2x}{x+y} - \frac{x^2 - y^2}{(x+y)^2}$

In[90]:= D[(x^2 - y^2) / (x + y), y]

Out[90]= $-\frac{2y}{x+y} - \frac{x^2 - y^2}{(x+y)^2}$

In[91]:= D[x y Exp[y^2], x]

Out[91]= $e^{y^2} y$

In[92]:= D[x y Exp[y^2], y]

Out[92]= $e^{y^2} x + 2 e^{y^2} x y^2$

In[93]:= D[(2 x - x y) / (u - v), x]

Out[93]= $\frac{2-y}{u-v}$

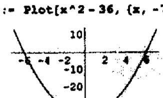
In[94]:= D[(2 x - x y) / (u - v), y]

Out[94]= $-\frac{x}{u-v}$

In[96]:= Solve[x^2 - 36 == 0, x]

Out[96]= {{x → -6}, {x → 6}}

In[97]:= Plot[x^2 - 36, {x, -7, 7}]

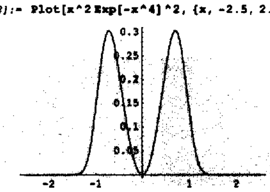


Out[97]= - Graphics -

In[3]:= Integrate[x^2 Exp[-x^4]^2, {x, -Infinity, Infinity}]

Out[3]= $\frac{\text{Gamma}[\frac{3}{4}]}{2\, 2^{3/4}}$

In[108]:= Plot[x^2 Exp[-x^4]^2, {x, -2.5, 2.5}, PlotRange → All]

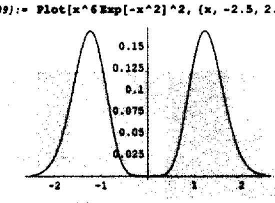


Out[108]= - Graphics -

In[110]:= Integrate[x^6 Exp[-x^2]^2, {x, -Infinity, Infinity}]

Out[110]= $\frac{15\sqrt{\frac{\pi}{2}}}{64}$

In[109]:= Plot[x^6 Exp[-x^2]^2, {x, -2.5, 2.5}, PlotRange → All]

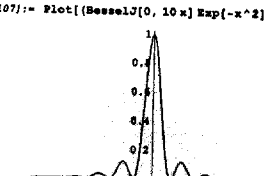


Out[109]= - Graphics -

In[9]:= Integrate[(BesselJ[0, 10 x] Exp[-x^2])^2, {x, -Infinity, Infinity}]

Out[9]= $\sqrt{\frac{\pi}{2}}$ HypergeometricPFQ[{$\frac{1}{2}$, $\frac{1}{2}$}, {1, 1}, -50]

In[107]:= Plot[(BesselJ[0, 10 x] Exp[-x^2])^2, {x, -1.5, 1.5}, PlotRange → All]



Out[107]= - Graphics -