Program of Quantum Mechanics

The Postulates

- **Postulate I**: The state of a system is defined by a wavefunction, ψ, which contains all the information that can be known about the system.
- **Postulate II**: Every physical observable is represented by a linear operator.
- **Postulate III**: The measurement of a physical observable will give a result that is one of the eigenvalues of the corresponding operator.

The Wavefunction

Criteria for a "good" wavefunction

- Single valueness
- continuous and finite
- continuous and finite first derivative
- $\int_{\text{space}} |\psi(x,y,z)|^2 \, dxdydz < \infty \underset{\text{wavefunction}}{\overset{\text{normalized}}{\Longrightarrow}} \int_{\text{space}} |\psi(x,y,z)|^2 \, dxdydz = 1$

The Quantum Mechanical Calculation

1. Define the classical Hamiltonian for the system (if possible).
2. Use *Postulate II* to replace the classical variables, x, p_x etc., with their appropriate operators.
3. Write the *Schrödinger equation*, $\hat{H}\psi = E\psi$,
4. Replace the operators $\hat{x}$, $\hat{p}_x$ etc. with x, $-i\hbar\frac{\partial}{\partial x}$
5. Solve the second order differential equation. Find the eigenvalues and eigenfunctions

Steps 1–4 give

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi = E\psi, \text{ (1D)}$$

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V(x,y,z)\psi = E\psi, \text{ (3D)}$$