Practice Quiz

1. What are the properties of a valid wavefunction?

2. Normalize $\psi(x) = x$, where all space is $0 \leq x \leq 1$.

3. Calculate the average value of momentum for a system described by the wavefunction in question 2. (Note your answer will be imaginary, but that is OK since the wavefunction is artificial.)

4. Label the following statements as true or false.

   (a) The operators corresponding to momentum and position are complementary

   (b) The wavefunction itself is interpreted as a probability.

   (c) Paul Dirac developed an approach to quantum mechanics called matrix mechanics

   (d) The Hamiltonian operator corresponds to energy.

   (e) Any quantum system has a classical analog from which one can construct the Hamiltonian operator.

Key

Practice Quiz

1. What are the properties of a valid wavefunction?

1) continuous and finite
2) continuous and finite 1st derivative
3) single valued
4) Normalizable

2. Normalize $\psi(x) = x$, where all space is $0 \le x \le 1$.

$$N = \sqrt{\int_0^1 \psi(x)^* \psi(x)\,dx} = \sqrt{\int_0^1 x^2\,dx} = \sqrt{\left.\frac{x^3}{3}\right|_0^1} = \sqrt{\frac{1}{3}}$$

so $\psi_{norm} = \sqrt{3}\,x$

3. Calculate the average value of momentum for a system described by the wavefunction in question 2. (Note your answer will be imaginary, but that is OK since the wavefunction is artificial.)

$$\langle \hat{p} \rangle = \int_0^1 x \left(-i\hbar \frac{\partial}{\partial x}\right) x\,dx = -i\hbar \int_0^1 x \frac{\partial x}{\partial x}\,dx = -i\hbar \int_0^1 x\,dx$$

$$= -i\hbar \left.\frac{x^2}{2}\right|_0^1 = \boxed{\frac{-i\hbar}{2}}$$

4. Label the following statements as true or false.

T (a) The operators corresponding to momentum and position are complementary

F (b) The wavefunction itself is interpreted as a probability.

F (c) Paul Dirac developed an approach to quantum mechanics called matrix mechanics

T (d) The Hamiltonian operator corresponds to energy.

F (e) Any quantum system has a classical analog from which one can construct the Hamiltonian operator.