

Directions: Answer the following questions on separate paper. You may use your book, notes and appropriate technology, but you must work independently. You may not receive help from any other individual (classmates, professors, or “virtual” tutors). If you do not show your work, you will receive no credit. The point value of each question is indicated. **IF YOU DO NOT FOLLOW DIRECTIONS YOU WILL BE PENALIZED!**

1. (20 points) Use the method of undetermined coefficients to determine the solution of the initial value problem

$$y'' - 2y' + y = xe^x + 4, \quad y(0) = 1, \quad y'(0) = 1$$

Simply using DSolve in *Mathematica* is not sufficient here; you must show all the steps by hand.

2. (20 points) Find the general solution of $xy'' - \frac{6y}{x} = 25x^2$. Again, simply using DSolve in *Mathematica* is not sufficient here; you must show all the steps by hand.
3. (20 points) In many physical systems the input function (that is, the nonhomogeneous term) may be specified by different formulas in different time periods. As an example, determine the solution of

$$y'' + y = \begin{cases} t & 0 \leq t \leq \pi, \\ \pi e^{\pi-t} & t > \pi. \end{cases}$$

satisfying the initial conditions $y(0) = 0$ and $y'(0) = 1$. Assume that y and y' are continuous at $t = \pi$. Plot the nonhomogeneous term and the solution as functions of time. (Hint: First solve the initial value problem for $t \leq \pi$; then solve for $t > \pi$.) Simply using DSolve in *Mathematica* is not sufficient here; you must show all the steps by hand.

4. A 2 kg mass is attached to a spring whose constant is 26 N/m. The surrounding medium offers a damping force numerically equal to 8 times the velocity. The mass is then released from an initial position 3 m below the equilibrium position with an upward velocity of 18 m/s.
- (a) (8 points) Determine the equation of motion.
 - (b) (3 points) Write the equation of motion in the form $x(t) = A \sin(\omega t - \phi)$.
 - (c) (2 points) What is the amplitude of the motion?
 - (d) (2 points) What is the period of the motion?
 - (e) (2 points) What is the frequency of the motion?
 - (f) (3 points) Determine the time(s) at which the mass crosses the equilibrium position.
5. (20 points) Find the eigenvalues and eigenfunctions for

$$x^2 y'' + xy' + \lambda y = 0, \quad y(1) = 0, \quad y(e^\pi) = 0$$