Practice Exam #1

Lenarz - Math 223 February 1, 2006 Name: ____________________

Directions: Answer all questions. You may use a calculator, but you must show your work. **If no work is shown, you will receive no credit**.

1. Find the standard form of the equation of the sphere given by $x^2+y^2+z^2-6x+2y-8z=0$. What is the center? What is the radius?

2. For each of the following (i) draw $\mathbf{u}$ and $\mathbf{v}$ with their initial point at the origin, (ii) graphically represent $\mathbf{u}+\mathbf{v}$, (iii) find $\mathbf{u}+\mathbf{v}$
   (a) $\mathbf{u}=2\mathbf{i}+\mathbf{j}$, $\mathbf{v}=-\mathbf{i}-3\mathbf{j}$
   (b) $\mathbf{u}=\mathbf{i}-\mathbf{j}$, $\mathbf{v}=-3\mathbf{i}$
   (c) $\mathbf{u}=5\mathbf{i}-4\mathbf{j}$, $\mathbf{v}=-2\mathbf{i}-3\mathbf{j}$

3. Find (i) $||\mathbf{u}||$, (ii) $||\mathbf{v}||$, (iii) a unit vector in the direction of $\mathbf{u}$, (iv) $\mathbf{u}\cdot\mathbf{v}$, (v) the angle between $\mathbf{u}$ and $\mathbf{v}$, (vi) the vector projection of $\mathbf{u}$ onto $\mathbf{v}$, (vii) the scalar projection of $\mathbf{u}$ onto $\mathbf{v}$
   (a) $\mathbf{u}=2\mathbf{i}+\mathbf{j}$, $\mathbf{v}=-\mathbf{i}-3\mathbf{j}$
   (b) $\mathbf{u}=\mathbf{i}-\mathbf{j}$, $\mathbf{v}=-3\mathbf{i}$
   (c) $\mathbf{u}=2\mathbf{i}-\mathbf{j}+2\mathbf{k}$, $\mathbf{v}=5\mathbf{i}+\mathbf{j}-3\mathbf{k}$

4. Find parametric equations of the line through the points $P_1(-2,1,5)$ and $P_2(6,2,-3)$.

5. Write the equation of the plane through the point $(-5,7,-2)$ that is
   (a) parallel to the xz-plane
   (b) perpendicular to the x-axis
   (c) parallel to both the x- and y-axes
   (d) parallel to the plane $3x-4y+z=7$

6. Find the smaller angle between the planes $2x-4y+z=7$ and $3x+2y-5z=9$.

7. Find the equation of the plane
   (a) containing the vectors $\mathbf{v}=3\mathbf{i}+3\mathbf{j}-\mathbf{k}$ and $\mathbf{u}=-\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ and passing through the point $(1,-1,0)$
   (b) containing the points $P_1(3,-6,4)$, $P_2(2,1,1)$, and $P_3(5,0,-2)$

8. For each equation, name the quadric surface and give a brief description (or sketch).
   (a) $x^2+y^2=81$
   (b) $x^2+y^2+z^2=81$
   (c) $z^2=4y$
   (d) $x^2+z^2=4y$
   (e) $3y-6z-12=0$

(f) $3x + 3y - 6z - 12 = 0$

(g) $x^2 + y^2 - z^2 - 1 = 0$

(h) $3x^2 + 4y^2 + 9z^2 - 36 = 0$

(i) $3x^2 + 4y^2 + 9z^2 + 36 = 0$

9. Convert the following from Cartesian to (i) cylindrical and (ii) spherical:
   (a) $x^2 + y^2 = 9$
   (b) $x^2 + 4y^2 = 16$
   (c) $x^2 + y^2 = 9z$
   (d) $x^2 + y^2 + 4z^2 = 10$

10. Convert the following from cylindrical to (i) Cartesian and (ii) spherical:
    (a) $r^2 + z^2 = 9$
    (b) $r^2 \cos^2\theta + z^2 = 4$
    (c) $r^2 \cos 2\theta + z^2 = 1$

11. Convert the following from spherical to (i) Cartesian and (ii) cylindrical:
    (a) $\rho = 4$
    (b) $\rho = 2\cos\phi$
    (c) $\rho \sin\phi = 1$
    (d) $\rho = \csc\phi \cot\phi$

12. The equations below are in rectangular, cylindrical or spherical coordinates. For each equation, describe it's three dimensional graph:
    (a) $r = 4\cos\theta$
    (b) $z = r$
    (c) $r = 4\sec\theta$
    (d) $\rho = 2\sec\phi$
    (e) $\rho = 1$
    (f) $\phi = \frac{\pi}{4}$

13. Find $\mathbf{r}'(t)$, $\mathbf{r}''(t)$.
    (a) $\mathbf{r}(t) = (\ln 3t^4)\mathbf{i} + (t^2 + 2t)\mathbf{j}$
    (b) $\mathbf{r}(t) = (\sin 2t)\mathbf{i} + (e^{3t+6})\mathbf{j}$

14. Find the equation of the line tangent to the curve at the point indicated.
    (a) $x(t) = 2t^3 - 4t + 7$, $y(t) = t + \ln(t + 1)$, $t = 1$

(b) $\mathbf{r}(t) = t\mathbf{i} + \frac{1}{2}t^2\mathbf{j} + \frac{1}{3}t^3\mathbf{k}$, $\left(2, 2, \frac{8}{3}\right)$

15. Given the acceleration vector $\mathbf{a}(t)$ of a point, the initial velocity, and initial position, find the velocity and position vectors of the point.

    (a) $\mathbf{a}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$, $\mathbf{v}(0) = 2\mathbf{i} - \mathbf{j}$, $\mathbf{r}(0) = -\mathbf{i} + 3\mathbf{j}$

    (b) $\mathbf{a}(t) = (3t^2)\mathbf{i} + (t^3)\mathbf{j}$, $\mathbf{v}(0) = \mathbf{i} + 2\mathbf{j}$, $\mathbf{r}(0) = 3\mathbf{i} - \mathbf{j}$