

Directions: Answer all questions in the space provided. Please circle your answer. The point value of each question is indicated in the margin.

(5) 1. $\int x \ln x \, dx$

Solution: Let

$$u = \ln x \qquad dv = x \, dx$$

so that

$$du = \frac{1}{x} dx \qquad v = \frac{1}{2}x^2$$

Then we have

$$\begin{aligned} \int x \ln x \, dx &= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \left(\frac{1}{x}\right) dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{2} \int x \, dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C \end{aligned}$$

(5) 2. $\int \tan^{-1} x \, dx$

Solution: Let

$$u = \tan^{-1} x \qquad dv = dx$$

so that

$$du = \frac{1}{1+x^2} dx \qquad v = x$$

Then we have

$$\begin{aligned} \int \tan^{-1} x \, dx &= x \tan^{-1} x - \int \frac{x}{1+x^2} dx \\ &= x \tan^{-1} x - \frac{1}{2} \int \frac{1}{t} dt \\ &= x \tan^{-1} x - \frac{1}{2} \ln |t| + C \\ &= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C \end{aligned}$$

where the substitution $t = 1 + x^2$, $dt = 2x \, dx$ was made in the second line.