Lenarz | Exam #2 Form B | Name: ____________
Math 121 | October 19, 2005 | Section: ____________

Directions: Answer the following questions in the space provided. You may not use a calculator. If you do not show your work, you will receive no credit. The point value of each question is indicated. **CIRCLE YOUR FINAL ANSWER. IF YOU DO NOT FOLLOW DIRECTIONS YOU WILL BE PENALIZED!**

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1. For each of the following functions $f(x)$, find the derivative $f'(x)$.

   (a) (5 points) $f(x) = 3^x$

   $$f'(x) = 3^x(\ln 3)$$

   (b) (5 points) $f(x) = (2x^3 - x^2 + 5)^{50}$

   $$f'(x) = 50(2x^3 - x^2 + 5)^{49}(6x^2 - 2x)$$

   (c) (5 points) $f(x) = 3x^3 + 2x^2 - 9x + 15$

   $$f'(x) = 9x^2 + 4x - 9$$

   (d) (5 points) $f(x) = e^3$
   Notice that e^3 is just a constant, so

   $$f'(x) = 0$$

   (e) (5 points) $f(x) = \tan x - \cos x$

   $$f'(x) = \sec^2 x - (-\sin x) = \sec^2 x + \sin x$$

   (f) (5 points) $f(x) = x \sin x$
   Need product rule

   $$f'(x) = x\cos x + \sin x$$

   (g) (5 points) $f(x) = \dfrac{e^x - 2x}{x^2 + 2}$
   Need quotient rule

   $$\begin{aligned} f'(x) &= \frac{(x^2+2)\frac{d}{dx}(e^x - 2x) - (e^x - 2x)\frac{d}{dx}(x^2+2)}{(x^2+2)^2} \\ &= \frac{(x^2+2)(e^x - 2) - (e^x - 2x)(2x)}{(x^2+2)^2} \end{aligned}$$

   (h) (5 points) $f(x) = \csc(x^2 + 2x)$
   Need chain rule

   $$\begin{aligned} f'(x) &= -\csc(x^2+2x)\cot(x^2+2x)\frac{d}{dx}(x^2+2x) \\ &= -(2x+2)\csc(x^2+2x)\cot(x^2+2x) \\ &= -2(x+1)\csc(x^2+2x)\cot(x^2+2x) \end{aligned}$$

(i) (5 points) $f(x) = \dfrac{x^2+3x+5}{\sqrt{x}}$

Notice that

$$f(x) = x^{\frac{3}{2}} + 3x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}$$

so we have

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + \frac{3}{2}x^{-\frac{1}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$$

(j) (5 points) $f(x) = \sin^{-1}(3x-1)$ Need chain rule

$$\begin{aligned} f'(x) &= \frac{1}{\sqrt{1-(3x-1)^2}}\frac{d}{dx}(3x-1) \\ &= \frac{3}{\sqrt{1+(3x-1)^2}} \end{aligned}$$

2. Suppose f and g are differentiable functions and that $f(1) = 0$, $g(1) = 1$, $f'(1) = -1$ and $g'(1) = 2$

(a) (5 points) If $h(x) = f(x) + g(x)$, find $h'(1)$

$$h'(x) = f'(x) + g'(x)$$

so we have

$$\begin{aligned} h'(1) &= f'(1) + g'(1) \\ &= (-1) + (2) \\ &= 1 \end{aligned}$$

(b) (5 points) If $H(x) = f(x)g(x)$, find $H'(1)$

$$H'(x) = f(x)g'(x) + f'(x)g(x)$$

so we have

$$\begin{aligned} H'(1) &= f(1)g'(1) + f'(1)g(1) \\ &= (0)(2) + (-1)(1) \\ &= -1 \end{aligned}$$

(c) (5 points) If $G(x) = \dfrac{f(x)}{g(x)}$, find $G'(1)$

$$G'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

so we have

$$\begin{aligned} G'(1) &= \frac{g(1)f'(1) - f(1)g'(1)}{[g(1)]^2} \\ &= \frac{(1)(-1) - (0)(2)}{(1)^2} \\ &= -1 \end{aligned}$$

(d) (5 points) If $F(x) = f(g(x))$, find $F'(1)$

$$F'(x) = f'(g(x))g'(x)$$

so we have

$$\begin{aligned} F'(1) &= f'(g(1))g'(1) \\ &= f'(1)g'(1) \\ &= (-1)(2) \\ &= -2 \end{aligned}$$

3. (10 points) The position of an object moving in a straight line is given by

$$f(t) = 2t^3 - 3t^2 - 12t$$

Find the velocity and acceleration of the object at time t.

$$v(t) = f'(t) = 6t^2 - 6t - 12$$

and

$$a(t) = f''(t) = 12t - 6$$

4. (5 points) Use implicit differentiation to find an equation for the line tangent to

$$y^3 - xy = 3$$

at the point $(-2, 1)$.

$$\begin{aligned} \frac{d}{dx}(y^3 - xy) &= \frac{d}{dx}(3) \\ 3y^2\frac{dy}{dx} - x\frac{dy}{dx} + y(-1) &= 0 \\ (3y^2 - x)\frac{dy}{dx} &= y \\ \frac{dy}{dx} &= \frac{y}{3y^2 - x} \end{aligned}$$

So the slope of our tangent line at $(-2, 1)$ is

$$m = \frac{(1)}{3(1)^2 - (-2)} = \frac{1}{5}$$

and the equation of the tangent line is

$$\begin{aligned} y - 1 &= \frac{1}{5}(x + 2) \\ y - 1 &= \frac{1}{5}x + \frac{2}{5} \\ y &= \frac{1}{5}x + \frac{7}{5} \end{aligned}$$

5. (10 points) Find a formula for $f^{(n)}(x)$ if

$$f(x) = \frac{1}{7-x}$$

Hint: find the first few derivatives and see if there is a pattern.

$$\begin{aligned}
f(x) &= (7-x)^{-1} \\
f'(x) &= (-1)(7-x)^{-2}(-1) \\
&= (7-x)^{-2} \\
f''(x) &= (-2)(7-x)^{-3}(-1) \\
&= 2(7-x)^{-3} \\
f'''(x) &= 2(-3)(7-x)^{-4}(-1) \\
&= 2(3)(7-x)^{-4} \\
f^{(4)}(x) &= 2(3)(-4)(7-x)^{-5}(-1) \\
&= 2(3)(4)(7-x)^{-5} \\
f^{(5)}(x) &= 2(3)(4)(-5)(7-x)^{-6}(-1) \\
&= 2(3)(4)(5)(7-x)^{-6}
\end{aligned}$$

So we would guess that

$$f^{(n)}(x) = 2(3)(4)(5)\ldots(n-1)(n)(7-x)^{-(n+1)} = n!(7-x)^{-(n+1)}$$

6. (5 points) Find the points at which the line tangent to

$$y = 3x^4 - 4x^3 - 12x^2 + 8$$

is horizontal.

The tangent line is horizontal when its slope is zero. Since the slope of the tangent line is given by

$$y' = 12x^3 - 12x^2 - 24x$$

we have

$$\begin{aligned}
0 &= 12x^3 - 12x^2 - 24x \\
&= 12x(x^2 - x - 2) \\
&= 12x(x-2)(x+1)
\end{aligned}$$

So $x = 0$, $x = 2$ or $x = -1$. If $x = 0$ then

$$\begin{aligned}
y &= 3(0)^4 - 4(0)^3 - 12(0)^2 + 8 \\
&= 8
\end{aligned}$$

So one point is $(0,8)$. If $x=2$ then

$$\begin{aligned} y &= 3(2)^4 - 4(2)^3 - 12(2)^2 + 8 \\ &= 3(16) - 4(8) - 12(4) + 8 \\ &= 48 - 32 - 48 + 8 \\ &= -24 \end{aligned}$$

So another point is $(2,-24)$. If $x=-1$, then

$$\begin{aligned} y &= 3(-1)^4 - 4(-1)^3 - 12(-1)^2 + 8 \\ &= 3 + 4 - 12 + 8 \\ &= 3 \end{aligned}$$

So the last point is $(-1,3)$.