1.

$$\begin{aligned} y &= (x^4 - 3x^2 + 5)^3 \\ y' &= 3(x^4 - 3x^2 + 5)^2 \frac{d}{dx}(x^4 - 3x^3 + 5) \\ &= 3(x^4 - 3x^2 + 5)^2 (4x^3 - 6x) \end{aligned}$$

2.

$$\begin{aligned} y &= \cos(\tan x) \\ y' &= -\sin(\tan x) \frac{d}{dx}(\tan x) \\ &= -\sin(\tan x) \sec^2 x \end{aligned}$$

3.

$$\begin{aligned} y &= \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} \\ &= x^{\frac{1}{2}} + x^{-\frac{4}{3}} \\ y' &= \frac{1}{2} x^{-\frac{1}{2}} - \frac{4}{3} x^{-\frac{7}{3}} \\ &= \frac{1}{2\sqrt{x}} - \frac{4}{3\sqrt[3]{x^7}} \end{aligned}$$

4.

$$
\begin{aligned}
y &= \frac{3x-2}{\sqrt{2x+1}} \\
&= \frac{3x-2}{(2x+1)^{\frac{1}{2}}} \\
y' &= \frac{(2x+1)^{\frac{1}{2}}\frac{d}{dx}(3x-2)-(3x-2)\frac{d}{dx}[(2x+1)^{\frac{1}{2}}]}{\left((2x+1)^{\frac{1}{2}}\right)^2} \\
&= \frac{(2x+1)^{\frac{1}{2}}(3)-(3x-2)\left[\frac{1}{2}(2x+1)^{-\frac{1}{2}}\frac{d}{dx}(2x+1)\right]}{2x+1} \\
&= \frac{3(2x+1)^{\frac{1}{2}}-(3x-2)\left[\frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)\right]}{2x+1} \\
&= \frac{3(2x+1)^{\frac{1}{2}}-(3x-2)(2x+1)^{-\frac{1}{2}}}{2x+1} \\
&= \frac{3(2x+1)-(3x-2)}{(2x+1)^{\frac{3}{2}}} \\
&= \frac{6x+3-3x+2}{(2x+1)^{\frac{3}{2}}} \\
&= \frac{3x+5}{(2x+1)^{\frac{3}{2}}}
\end{aligned}
$$

5.

$$
\begin{aligned}
y &= 2x\sqrt{x^2+1} \\
&= 2x(x^2+1)^{\frac{1}{2}} \\
y' &= 2x\frac{d}{dx}[(x^2+1)^{\frac{1}{2}}]+(x^2+1)^{\frac{1}{2}}\frac{d}{dx}(2x) \\
&= 2x\left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\frac{d}{dx}(x^2+1)\right]+(x^2+1)^{\frac{1}{2}}(2) \\
&= x[(x^2+1)^{-\frac{1}{2}}(2x)]+2(x^2+1)^{\frac{1}{2}} \\
&= 2x^2(x^2+1)^{-\frac{1}{2}}+2(x^2+1)^{\frac{1}{2}} \\
&= \frac{2x^2}{\sqrt{x^2+1}}+2\sqrt{x^2+1} \\
&= \frac{2x^2+2\sqrt{x^2+1}}{\sqrt{x^2+1}}
\end{aligned}
$$

6.

$$\begin{aligned}
y &= \frac{e^x}{1+x^2} \\
y' &= \frac{(1+x^2)\dfrac{d}{dx}(e^x) - (e^x)\dfrac{d}{dx}(1+x^2)}{(1+x^2)^2} \\
&= \frac{(1+x^2)(e^x) - (e^x)(2x)}{(1+x^2)^2} \\
&= \frac{e^x(1+x^2-2x)}{(1+x^2)^2} \\
&= \frac{e^x(x-1)^2}{(1+x^2)^2}
\end{aligned}$$

7.

$$\begin{aligned}
y &= e^{\sin 2\theta} \\
y' &= e^{\sin 2\theta}\frac{d}{d\theta}(\sin 2\theta) \\
&= e^{\sin 2\theta}\left(\cos 2\theta \frac{d}{d\theta}(2\theta)\right) \\
&= e^{\sin 2\theta}(\cos 2\theta)(2) \\
&= (2\cos 2\theta)e^{\sin 2\theta}
\end{aligned}$$

8.

$$\begin{aligned}
y &= e^{-t}(t^2-2t+2) \\
y' &= e^{-t}\frac{d}{dt}(t^2-2t+2) + (t^2-2t+2)\frac{d}{dt}(e^{-t}) \\
&= e^{-t}(2t-2) + (t^2-2t+2)e^{-t}\frac{d}{dt}(-t) \\
&= e^{-t}(2t-2) + (t^2-2t+2)e^{-t}(-1) \\
&= e^{-t}(2t-2) - (t^2-2t+2)e^{-t} \\
&= e^{-t}((2t-2) - (t^2-2t+2)) \\
&= e^{-t}(2t-2-t^2+2t-2) \\
&= e^{-t}(-t^2+4t-4)
\end{aligned}$$

9.

$$\begin{aligned}
y &= \frac{t}{1-t^2} \\
y' &= \frac{(1-t^2)\frac{d}{dt}(t) - (t)\frac{d}{dt}(1-t^2)}{(1-t^2)^2} \\
&= \frac{(1-t^2)(1) - (t)(-2t)}{(1-t^2)^2} \\
&= \frac{1-t^2+2t^2}{(1-t^2)^2} \\
&= \frac{1+t^2}{(1-t^2)^2}
\end{aligned}$$

10.

$$\begin{aligned}
y &= \sin^{-1}(e^x) \\
y' &= \frac{1}{\sqrt{1-(e^x)^2}}\frac{d}{dx}(e^x) \\
&= \frac{1}{\sqrt{1-e^{2x}}}(e^x) \\
&= \frac{e^x}{\sqrt{1-e^{2x}}}
\end{aligned}$$

11.

$$\begin{aligned}
y &= xe^{-\frac{1}{x}} \\
y' &= x\frac{d}{dx}(e^{-\frac{1}{x}}) + (e^{-\frac{1}{x}})\frac{d}{dx}(x) \\
&= x\left(e^{-\frac{1}{x}}\frac{d}{dx}\left(-\frac{1}{x}\right)\right) + (e^{-\frac{1}{x}})(1) \\
&= x\left(e^{-\frac{1}{x}}\frac{d}{dx}\left(-x^{-1}\right)\right) + e^{-\frac{1}{x}} \\
&= x\left(e^{-\frac{1}{x}}\left(x^{-2}\right)\right) + e^{-\frac{1}{x}} \\
&= x^{-1}e^{-\frac{1}{x}} + e^{-\frac{1}{x}} \\
&= e^{-\frac{1}{x}}(x^{-1}+1) \\
&= e^{-\frac{1}{x}}\left(\frac{1}{x}+1\right)
\end{aligned}$$

12.

$$\begin{aligned}
y &= x^r e^{sx} \\
y' &= x^r \frac{d}{dx}(e^{sx}) + (e^{sx}) \frac{d}{dx}(x^r) \\
&= x^r \left(e^{sx} \frac{d}{dx}(sx) \right) + (e^{sx})(rx^{r-1}) \\
&= x^r \left(e^{sx}(s) \right) + rx^{r-1} e^{sx} \\
&= sx^r e^{sx} + rx^{r-1} e^{sx} \\
&= x^{r-1} e^{sx}(sx + r)
\end{aligned}$$

13.

$$\begin{aligned}
y &= \tan \sqrt{1-x} \\
&= \tan(1-x)^{\frac{1}{2}} \\
y' &= \sec^2(1-x)^{\frac{1}{2}} \frac{d}{dx}[(1-x)^{\frac{1}{2}}] \\
&= \sec^2(1-x)^{\frac{1}{2}} \left[\frac{1}{2}(1-x)^{-\frac{1}{2}} \frac{d}{dx}(1-x) \right] \\
&= \sec^2(1-x)^{\frac{1}{2}} \left[\frac{1}{2}(1-x)^{-\frac{1}{2}}(-1) \right] \\
&= -\frac{\sec^2(1-x)^{\frac{1}{2}}}{2(1-x)^{\frac{1}{2}}} \\
&= -\frac{\sec^2 \sqrt{1-x}}{2\sqrt{1-x}}
\end{aligned}$$

14.

$$\begin{aligned}
y &= \frac{1}{\sin(x - \sin x)} \\
&= [\sin(x - \sin x)]^{-1} \\
y' &= -[\sin(x - \sin x)]^{-2} \frac{d}{dx}[\sin(x - \sin x)] \\
&= -[\sin(x - \sin x)]^{-2}[\cos(x - \sin x) \frac{d}{dx}(x - \sin x)] \\
&= -[\sin(x - \sin x)]^{-2} \cos(x - \sin x)(1 - \cos x) \\
&= -\frac{(1 - \cos x)\cos(x - \sin x)}{[\sin(x - \sin x)]^2}
\end{aligned}$$

15. Use implicit differentiation

$$\begin{aligned}
xy^4 + x^2y &= x + 3y \\
\frac{d}{dx}(xy^4 + x^2y) &= \frac{d}{dx}(x + 3y) \\
x\frac{d}{dx}(y^4) + y^4\frac{d}{dx}(x) + x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2) &= 1 + 3\frac{dy}{dx} \\
x\left(4y^3\frac{dy}{dx}\right) + y^4(1) + x^2\frac{dy}{dx} + y(2x) &= 1 + 3\frac{dy}{dx} \\
4xy^3\frac{dy}{dx} + x^2\frac{dy}{dx} - 3\frac{dy}{dx} &= 1 - y^4 - 2xy \\
(4xy^3 + x^2 - 3)\frac{dy}{dx} &= 1 - y^4 - 2xy \\
\frac{dy}{dx} &= \frac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}
\end{aligned}$$

16. skip

17.

$$\begin{aligned}
y &= \frac{\sec 2\theta}{1 + \tan 2\theta} \\
y' &= \frac{(1 + \tan 2\theta)\frac{d}{d\theta}(\sec 2\theta) - (\sec 2\theta)\frac{d}{d\theta}(1 + \tan 2\theta)}{(1 + \tan 2\theta)^2} \\
&= \frac{(1 + \tan 2\theta)\left(\sec 2\theta \tan 2\theta \frac{d}{d\theta}(2\theta)\right) - (\sec 2\theta)\left(\sec^2 2\theta \frac{d}{d\theta}(2\theta)\right)}{(1 + \tan 2\theta)^2} \\
&= \frac{(1 + \tan 2\theta)(\sec 2\theta \tan 2\theta(2)) - (\sec 2\theta)(\sec^2 2\theta(2))}{(1 + \tan 2\theta)^2} \\
&= \frac{2\sec 2\theta \tan 2\theta + 2\sec 2\theta \tan^2 2\theta - 2\sec^3 2\theta}{(1 + \tan 2\theta)^2} \\
&= \frac{2\sec 2\theta(\tan 2\theta + \tan^2 2\theta - \sec^2 2\theta)}{(1 + \tan 2\theta)^2} \\
&= \frac{2\sec 2\theta(\tan 2\theta - 1)}{(1 + \tan 2\theta)^2}
\end{aligned}$$

The last step uses the fact that $\tan^2\theta + 1 = \sec^2\theta$.

18. Use implicit differentiation

$$\begin{aligned}
x^2\cos y + \sin 2y &= xy \\
\frac{d}{dx}(x^2\cos y + \sin 2y) &= \frac{d}{dx}(xy) \\
x^2\frac{d}{dx}(\cos y) + \cos y\frac{d}{dx}(x^2) + \frac{d}{dx}(\sin 2y) &= x\frac{d}{dx}(y) + y\frac{d}{dx}(x) \\
x^2(-\sin y)\frac{dy}{dx} + \cos y(2x) + \cos 2y\frac{d}{dx}(2y) &= x\frac{dy}{dx} + y(1) \\
-x^2\sin y\frac{dy}{dx} + 2x\cos y + \cos 2y\left(2\frac{dy}{dx}\right) &= x\frac{dy}{dx} + y \\
-x^2\sin y\frac{dy}{dx} + 2\cos 2y\frac{dy}{dx} - x\frac{dy}{dx} &= y - 2x\cos y \\
(-x^2\sin y + 2\cos 2y - x)\frac{dy}{dx} &= y - 2x\cos y \\
\frac{dy}{dx} &= \frac{y - 2x\cos y}{-x^2\sin y + 2\cos 2y - x}
\end{aligned}$$

19.

$$\begin{aligned}
y &= e^{cx}(c\sin x - \cos x) \\
y' &= e^{cx}\frac{d}{dx}(c\sin x - \cos x) + (c\sin x - \cos x)\frac{d}{dx}(e^{cx}) \\
&= e^{cx}(c\cos x - (-\sin x)) + (c\sin x - \cos x)e^{cx}\frac{d}{dx}(cx) \\
&= e^{cx}(c\cos x + \sin x) + (c\sin x - \cos x)e^{cx}(c) \\
&= e^{cx}[(c\cos x + \sin x) + c(c\sin x - \cos x)] \\
&= e^{cx}(c\cos x + \sin x + c^2\sin x - c\cos x) \\
&= e^{cx}(1 + c^2)\sin x
\end{aligned}$$

20. skip

21.

$$\begin{aligned}
y &= e^{e^x} \\
y' &= e^{e^x}\frac{d}{dx}(e^x) \\
&= e^{e^x}e^x \\
&= e^{e^x + x}
\end{aligned}$$

22.

$$\begin{aligned} y &= \sec(1+x^2) \\ y' &= \sec(1+x^2)\tan(1+x^2)\frac{d}{dx}(1+x^2) \\ &= \sec(1+x^2)\tan(1+x^2)(2x) \\ &= 2x\sec(1+x^2)\tan(1+x^2) \end{aligned}$$

23.

$$\begin{aligned} y &= (1-x^{-1})^{-1} \\ y' &= -(1-x^{-1})^{-2}\frac{d}{dx}(1-x^{-1}) \\ &= -(1-x^{-1})^{-2}(x^{-2}) \\ &= -x^{-2}(1-x^{-1}) \end{aligned}$$

24.

$$\begin{aligned} y &= \frac{1}{\sqrt[3]{x+\sqrt{x}}} \\ &= (x+x^{\frac{1}{2}})^{-\frac{1}{3}} \\ y' &= -\frac{1}{3}(x+x^{\frac{1}{2}})^{-\frac{4}{3}}\frac{d}{dx}(x+x^{\frac{1}{2}}) \\ &= -\frac{1}{3}(x+x^{\frac{1}{2}})^{-\frac{4}{3}}\left(1+\frac{1}{2}x^{-\frac{1}{2}}\right) \\ &= \left(-\frac{1}{3\sqrt[3]{(x+\sqrt{x})^4}}\right)\left(1+\frac{1}{2\sqrt{x}}\right) \end{aligned}$$

25. Use implicit differentiation

$$
\begin{aligned}
\sin(xy) &= x^2 - y \\
\frac{d}{dx}[\sin(xy)] &= \frac{d}{dx}(x^2 - y) \\
\cos(xy)\frac{d}{dx}(xy) &= 2x - \frac{dy}{dx} \\
\cos(xy)\left(x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right) &= 2x - \frac{dy}{dx} \\
\cos(xy)\left(x\frac{dy}{dx} + y(1)\right) &= 2x - \frac{dy}{dx} \\
x\cos(xy)\frac{dy}{dx} + y\cos(xy) &= 2x - \frac{dy}{dx} \\
x\cos(xy)\frac{dy}{dx} + \frac{dy}{dx} &= 2x - y\cos(xy) \\
\left(x\cos(xy) + 1\right)\frac{dy}{dx} &= 2x - y\cos(xy) \\
\frac{dy}{dx} &= \frac{2x - y\cos(xy)}{x\cos(xy) + 1}
\end{aligned}
$$

26.

$$
\begin{aligned}
y &= \sqrt{\sin\sqrt{x}} \\
&= (\sin x^{\frac{1}{2}})^{\frac{1}{2}} \\
y' &= \frac{1}{2}(\sin x^{\frac{1}{2}})^{-\frac{1}{2}}\frac{d}{dx}(\sin x^{\frac{1}{2}}) \\
&= \frac{1}{2}(\sin x^{\frac{1}{2}})^{-\frac{1}{2}}\left(\cos x^{\frac{1}{2}}\frac{d}{dx}(x^{\frac{1}{2}})\right) \\
&= \frac{1}{2}(\sin x^{\frac{1}{2}})^{-\frac{1}{2}}(\cos x^{\frac{1}{2}})\left(\frac{1}{2}x^{-\frac{1}{2}}\right) \\
&= \frac{\cos\sqrt{x}}{4\sqrt{x\sin\sqrt{x}}}
\end{aligned}
$$

27. skip

28. skip

29. skip

30. skip

31.

$$\begin{aligned}
y &= x\tan^{-1}(4x)\\
y' &= x\frac{d}{dx}[\tan^{-1}(4x)] + \tan^{-1}(4x)\frac{d}{dx}(x)\\
&= x\left(\frac{1}{1+(4x)^2}\right)\frac{d}{dx}(4x) + \tan^{-1}(4x)(1)\\
&= x\left(\frac{1}{1+16x^2}\right)(4) + \tan^{-1}(4x)\\
&= \frac{4x}{1+16x^2} + \tan^{-1}(4x)
\end{aligned}$$

32.

$$\begin{aligned}
y &= e^{\cos x} + \cos(e^x)\\
y' &= e^{\cos x}\frac{d}{dx}(\cos x) + (-\sin(e^x))\frac{d}{dx}(e^x)\\
&= e^{\cos x}(-\sin x) - \sin(e^x)(e^x)\\
&= -e^{\cos x}\sin x - e^x\sin(e^x)
\end{aligned}$$

33. skip

34.

$$\begin{aligned}
y &= 10^{\tan\pi\theta}\\
y' &= 10^{\tan\pi\theta}(\ln 10)\frac{d}{d\theta}(\tan\pi\theta)\\
&= 10^{\tan\pi\theta}(\ln 10)(\sec^2\pi\theta)\frac{d}{d\theta}(\pi\theta)\\
&= 10^{\tan\pi\theta}(\ln 10)(\sec^2\pi\theta)(\pi)\\
&= \pi 10^{\tan\pi\theta}\ln 10\sec^2\pi\theta
\end{aligned}$$

35.

$$\begin{aligned}
y &= \cot(3x^2+5)\\
y' &= -\csc^2(3x^2+5)\frac{d}{dx}(3x^2+5)\\
&= -\csc^2(3x^2+5)(6x)\\
&= -6x\csc^2(3x^2+5)
\end{aligned}$$

36. skip

37.

$$\begin{aligned}
y &= \sin(\tan\sqrt{1+x^3}) \\
&= \sin(\tan(1+x^3)^{\frac{1}{2}}) \\
y' &= \cos(\tan(1+x^3)^{\frac{1}{2}})\frac{d}{dx}(\tan(1+x^3)^{\frac{1}{2}}) \\
&= \cos(\tan(1+x^3)^{\frac{1}{2}})\left(\sec^2(1+x^3)^{\frac{1}{2}}\frac{d}{dx}(1+x^3)^{\frac{1}{2}}\right) \\
&= \cos(\tan(1+x^3)^{\frac{1}{2}})\left(\sec^2(1+x^3)^{\frac{1}{2}}\frac{1}{2}(1+x^3)^{-\frac{1}{2}}\frac{d}{dx}(1+x^3)\right) \\
&= \cos(\tan(1+x^3)^{\frac{1}{2}})\left(\sec^2(1+x^3)^{\frac{1}{2}}\frac{1}{2}(1+x^3)^{-\frac{1}{2}}(3x^2)\right) \\
&= \frac{3x^2\cos(\tan\sqrt{1+x^3})\sec^2\sqrt{1+x^3}}{2\sqrt{1+x^3}}
\end{aligned}$$

38.

$$\begin{aligned}
y &= \arctan(\arcsin\sqrt{x}) \\
&= \arctan(\arcsin x^{\frac{1}{2}}) \\
y' &= \frac{1}{1+(\arcsin x^{\frac{1}{2}})^2}\ \frac{d}{dx}(\arcsin x^{\frac{1}{2}}) \\
&= \frac{1}{1+(\arcsin x^{\frac{1}{2}})^2}\ \left(\frac{1}{\sqrt{1-(x^{\frac{1}{2}})^2}}\ \frac{d}{dx}(x^{\frac{1}{2}})\right) \\
&= \frac{1}{1+(\arcsin x^{\frac{1}{2}})^2}\ \left(\frac{1}{\sqrt{1-x}}\ \frac{1}{2}x^{-\frac{1}{2}}\right) \\
&= \frac{1}{2(1+(\arcsin\sqrt{x})^2)\sqrt{x(1-x)}}
\end{aligned}$$

39.

$$\begin{aligned}
y &= \tan^2(\sin\theta) \\
&= (\tan(\sin\theta))^2 \\
y' &= 2(\tan(\sin\theta))\frac{d}{d\theta}(\tan(\sin\theta)) \\
&= 2(\tan(\sin\theta))\sec^2(\sin\theta)\frac{d}{d\theta}(\sin\theta) \\
&= 2(\tan(\sin\theta))\sec^2(\sin\theta)(\cos\theta)
\end{aligned}$$

40. Use implicit differentiation

$$\begin{aligned}
xe^y &= y-1\\
\frac{d}{dx}(xe^y) &= \frac{d}{dx}(y-1)\\
x\frac{d}{dx}(e^y)+e^y\frac{d}{dx}(x) &= \frac{dy}{dx}\\
xe^y\frac{dy}{dx}+e^y(1) &= \frac{dy}{dx}\\
e^y &= \frac{dy}{dx}-xe^y\frac{dy}{dx}\\
e^y &= (1-xe^y)\frac{dy}{dx}\\
\frac{e^y}{1-xe^y} &= \frac{dy}{dx}
\end{aligned}$$

41. skip

42.

$$\begin{aligned}
y &= \frac{(x+\lambda)^4}{x^4\lambda^4}\\
y' &= \frac{(x^4+\lambda^4)\frac{d}{dx}(x+\lambda)^4-(x+\lambda)^4\frac{d}{dx}(x^4+\lambda^4)}{(x^4+\lambda^4)^2}\\
&= \frac{(x^4+\lambda^4)4(x+\lambda)^3\frac{d}{dx}(x+\lambda)-(x+\lambda)^4(4x^3)}{(x^4+\lambda^4)^2}\\
&= \frac{(x^4+\lambda^4)4(x+\lambda)^3(1)-(x+\lambda)^4(4x^3)}{(x^4+\lambda^4)^2}\\
&= \frac{4(x^4+\lambda^4)(x+\lambda)^3-4x^3(x+\lambda)^4}{(x^4+\lambda^4)^2}\\
&= \frac{4(x+\lambda)^3((x^4+\lambda^4)-x^3(x+\lambda)}{(x^4+\lambda^4)^2}\\
&= \frac{4(x+\lambda)^3(\lambda^4-x^3\lambda)}{(x^4+\lambda^4)^2}\\
&= \frac{4\lambda(x+\lambda)^3(\lambda^3-x^3)}{(x^4+\lambda^4)^2}
\end{aligned}$$

43. skip

44.

$$\begin{aligned}
y &= \frac{\sin mx}{x} \\
y' &= \frac{x\dfrac{d}{dx}(\sin mx) - (\sin mx)\dfrac{d}{dx}(x)}{x^2} \\
&= \frac{x(\cos mx)\dfrac{d}{dx}(mx) - (\sin mx)(1)}{x^2} \\
&= \frac{x(\cos mx)(m) - \sin mx}{x^2} \\
&= \frac{mx\cos mx - \sin mx}{x^2}
\end{aligned}$$

45. skip

46. skip

47. skip

48. skip

49. If $f(t) = \sqrt{4t+1}$ find $f''(2)$.

$$\begin{aligned}
f(t) &= (4t+1)^{\frac{1}{2}} \\
f'(t) &= \frac{1}{2}(4t+1)^{-\frac{1}{2}}\frac{d}{dt}(4t+1) \\
&= \frac{1}{2}(4t+1)^{-\frac{1}{2}}(4) \\
&= 2(4t+1)^{-\frac{1}{2}} \\
f''(t) &= 2\left(-\frac{1}{2}(4t+1)^{-\frac{3}{2}}\frac{d}{dt}(4t+1)\right) \\
&= 2\left(-\frac{1}{2}(4t+1)^{-\frac{3}{2}}(4)\right) \\
&= -4(4t+1)^{-\frac{3}{2}} \\
&= -\frac{4}{\sqrt{(4t+1)^3}} \\
f''(2) &= -\frac{4}{\sqrt{(4(2)+1)^3}} \\
&= -\frac{4}{\sqrt{(9)^3}} \\
&= -\frac{4}{27}
\end{aligned}$$

50. If $g(\theta) = \theta \sin\theta$, find $g''\left(\frac{\pi}{6}\right)$

$$
\begin{aligned}
g'(\theta) &= \theta \frac{d}{d\theta}(\sin\theta) + (\sin\theta)\frac{d}{d\theta}(\theta) \\
&= \theta\cos\theta + \sin\theta \\
g''(\theta) &= \theta \frac{d}{d\theta}\cos\theta + (\cos\theta)\frac{d}{d\theta}(\theta) + \cos\theta \\
&= \theta(-\sin\theta) + \cos\theta + \cos\theta \\
&= -\theta\sin\theta + 2\cos\theta \\
g''\left(\frac{\pi}{6}\right) &= -\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{6}\right) + 2\cos\left(\frac{\pi}{6}\right) \\
&= -\frac{\pi}{6}\left(\frac{1}{2}\right) + 2\left(\frac{\sqrt{3}}{2}\right) \\
&= -\frac{\pi}{12} + \sqrt{3}
\end{aligned}
$$

51. Find y'' if $x^6 + y^6 = 1$. Use implicit differentiation

$$\begin{aligned}
\frac{d}{dx}(x^6 + y^6) &= \frac{d}{dx}(1) \\
6x^5 + 6y^5\frac{dy}{dx} &= 0 \\
6y^5\frac{dy}{dx} &= -6x^5 \\
\frac{dy}{dx} &= -\frac{x^5}{y^5} \\
\frac{d}{dx}\left(\frac{dy}{dx}\right) &= \frac{d}{dx}\left(-\frac{x^5}{y^5}\right) \\
\frac{d^2y}{dx^2} &= \frac{y^5\frac{d}{dx}(-x^5) - (-x^5)\frac{d}{dx}(y^5)}{(y^5)^2} \\
&= \frac{y^5(-5x^4) + x^5\left(5y^4\frac{dy}{dx}\right)}{y^{10}} \\
&= \frac{-5x^4y^5 + 5x^5y^4\left(-\frac{x^5}{y^5}\right)}{y^{10}} \\
&= \frac{-5x^4y^5 - \frac{5x^{10}}{y}}{y^{10}} \\
&= \frac{\frac{-5x^4y^6 - 5x^{10}}{y}}{y^{10}} \\
&= \frac{-5x^4(y^6 + x^6)}{y^{11}} \\
&= \frac{-5x^4(1)}{y^{11}} \\
&= -\frac{5x^4}{y^{11}}
\end{aligned}$$

52. Find $f^{(n)}(x)$ if $f(x) = \dfrac{1}{2-x}$

$$\begin{aligned}
f(x) &= (2-x)^{-1} \\
f'(x) &= -(2-x)^{-2}(-1) \\
&= (2-x)^{-2} \\
f''(x) &= -2(2-x)^{-3}(-1) \\
&= 2(2-x)^{-3} \\
f'''(x) &= 2(-3)(2-x)^{-4}(-1) \\
&= 2 \cdot 3(2-x)^{-4} \\
f^{(4)}(x) &= 2 \cdot 3(-4)(2-x)^{-5}(-1) \\
&= 2 \cdot 3 \cdot 4(2-x)^{-5} \\
f^{(5)}(x) &= 2 \cdot 3 \cdot 4(-5)(2-x)^{-6}(-1) \\
&= 2 \cdot 3 \cdot 4 \cdot 5(2-x)^{-6} \\
f^{(n)}(x) &= 2 \cdot 3 \cdot 4 \cdot 5 \cdots n(2-x)^{-(n+1)} \\
&= n!(2-x)^{-n-1}
\end{aligned}$$

53. skip

54. skip

55.

$$\begin{aligned}
y &= 4\sin^2 x \\
&= 4(\sin x)^2 \\
y' &= 4(2(\sin x)\frac{d}{dx}(\sin x)) \\
&= 8 \sin x \cos x
\end{aligned}$$

So the slope of the tangent line is

$$\begin{aligned}
m &= 8 \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) \\
&= 8\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \\
&= 2\sqrt{3}
\end{aligned}$$

And the equation of the tangent line is

$$\begin{aligned}
y - 1 &= 2\sqrt{3}\left(x - \frac{\pi}{6}\right) \\
y - 1 &= 2\sqrt{3}x - \frac{\pi\sqrt{3}}{3} \\
y &= 2\sqrt{3}x + 1 - \frac{\pi\sqrt{3}}{3}
\end{aligned}$$

56.

$$\begin{aligned}
y &= \frac{x^2-1}{x^2+1} \\
y' &= \frac{(x^2+1)\frac{d}{dx}(x^2-1)-(x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2} \\
&= \frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2} \\
&= \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} \\
&= \frac{4x}{(x^2+1)^2}
\end{aligned}$$

So the slope of the tangent line is

$$\begin{aligned}
m &= \frac{4(0)}{((0)^2+1)^2} \\
&= \frac{0}{1} \\
&= 0
\end{aligned}$$

And the equation of the tangent line is

$$\begin{aligned}
y-(-1) &= (0)(x-0) \\
y+1 &= 0 \\
y &= -1
\end{aligned}$$

57.

$$\begin{aligned}
y &= \sqrt{1+4\sin x} \\
&= (1+4\sin x)^{\frac{1}{2}} \\
y' &= \frac{1}{2}(1+4\sin x)^{-\frac{1}{2}}\frac{d}{dx}(1+4\sin x) \\
&= \frac{1}{2}(1+4\sin x)^{-\frac{1}{2}}(4\cos x) \\
&= 2\cos x(1+4\sin x)^{-\frac{1}{2}} \\
&= \frac{2\cos x}{\sqrt{1+4\sin x}}
\end{aligned}$$

So the slope of the tangent line is

$$\begin{aligned}
m &= \frac{2\cos(0)}{\sqrt{1+4\sin(0)}} \\
&= \frac{2}{1} \\
&= 2
\end{aligned}$$

And the equation of the tangent line is

$$\begin{aligned} y - 1 &= 2(x - 0) \\ y - 1 &= 2x \\ y &= 2x + 1 \end{aligned}$$

58. Use implicit differentiation

$$\begin{aligned} \frac{d}{dx}(x^2 + 4xy + y^2) &= \frac{d}{dx}(13) \\ 2x + 4x\frac{d}{dx}(y) + y\frac{d}{dx}(4x) + 2y\frac{dy}{dx} &= 0 \\ 2x + 4x\frac{dy}{dx} + 4y + 2y\frac{dy}{dx} &= 0 \\ 4x\frac{dy}{dx} + 2y\frac{dy}{dx} &= -2x - 4y \\ (4x + 2y)\frac{dy}{dx} &= -2x - 4y \\ \frac{dy}{dx} &= \frac{-2x - 4y}{4x + 2y} \\ &= \frac{-x - 2y}{2x + y} \end{aligned}$$

So the slope of the tangent line is

$$\begin{aligned} m &= \frac{-(2) - 2(1)}{2(2) + (1)} \\ &= \frac{-4}{5} \\ &= -\frac{4}{5} \end{aligned}$$

And the equation of the tangent line is

$$\begin{aligned} y - 1 &= -\frac{4}{5}(x - 2) \\ y - 1 &= -\frac{4}{5}x + \frac{8}{5} \\ y &= -\frac{4}{5}x + \frac{13}{5} \end{aligned}$$

59.

$$\begin{aligned}
y &= (2+x)e^{-x} \\
y' &= (2+x)\frac{d}{dx}(e^{-x}) + (e^{-x})\frac{d}{dx}(2+x) \\
&= (2+x)e^{-x}\frac{d}{dx}(-x) + e^{-x}(1) \\
&= (2+x)e^{-x}(-1) + e^{-x} \\
&= (-2-x)e^{-x} + e^{-x} \\
&= e^{-x}(-2-x+1) \\
&= e^{-x}(-1-x)
\end{aligned}$$

So the slope of the tangent line is

$$\begin{aligned}
m &= e^{-(0)}(-1-(0)) \\
&= (1)(-1) \\
&= -1
\end{aligned}$$

And the equation of the tangent line is

$$\begin{aligned}
y-2 &= (-1)(x-0) \\
y-2 &= -x \\
y &= -x+2
\end{aligned}$$

60. skip

61. (a)

$$\begin{aligned}
f(x) &= x\sqrt{5-x} \\
&= x(5-x)^{\frac{1}{2}} \\
f'(x) &= x\frac{d}{dx}(5-x)^{\frac{1}{2}} + (5-x)^{\frac{1}{2}}\frac{d}{dx}(x) \\
&= x\left(\frac{1}{2}(5-x)^{-\frac{1}{2}}\frac{d}{dx}(5-x)\right) + (5-x)^{\frac{1}{2}}(1) \\
&= x\left(\frac{1}{2}(5-x)^{-\frac{1}{2}}(-1)\right) + (5-x)^{\frac{1}{2}} \\
&= \frac{-x}{2\sqrt{5-x}} + \sqrt{5-x} \\
&= \frac{-x+2(5-x)}{2\sqrt{5-x}} \\
&= \frac{10-3x}{2\sqrt{5-x}}
\end{aligned}$$

(b) At the point $(1, 2)$, the slope of the tangent line is

$$\begin{aligned} m &= \frac{10-3(1)}{2\sqrt{5-(1)}} \\ &= \frac{7}{4} \end{aligned}$$

So the equation of the line tangent to f at $(1, 2)$ is

$$\begin{aligned} y-2 &= \frac{7}{4}(x-1) \\ y-2 &= \frac{7}{4}x-\frac{7}{4} \\ y &= \frac{7}{4}x+\frac{1}{4} \end{aligned}$$

At the point $(4, 4)$, the slope of the tangent line is

$$\begin{aligned} m &= \frac{10-3(4)}{2\sqrt{5-(4)}} \\ &= \frac{-2}{2} \\ &= -1 \end{aligned}$$

So the equation of the line tangent to f at $(4, 4)$ is

$$\begin{aligned} y-4 &= (-1)(x-4) \\ y-4 &= -x+4 \\ y &= -x+8 \end{aligned}$$

(c) skip

(d) skip

62. skip

63. skip

64. Find the points on the ellipse $x^2 + 2y^2 = 1$ where the tangent line has slope 1. Use implicit differentiation

$$\begin{aligned} x^2+2y^2 &= 1 \\ \frac{d}{dx}(x^2+2y^2) &= \frac{d}{dx}(1) \\ 2x+4y\frac{dy}{dx} &= 0 \\ 4y\frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= -\frac{x}{2y} \end{aligned}$$

To find points at which the slope is 1, we set $\frac{dy}{dx} = 1$

$$\begin{aligned} 1 &= -\frac{x}{2y} \\ 2y &= -x \\ -2y &= x \end{aligned}$$

If we substitute this back into the original equation, we will have

$$\begin{aligned} (-2y)^2 + 2y^2 &= 1 \\ 4y^2 + 2y^2 &= 1 \\ 6y^2 &= 1 \\ y^2 &= \frac{1}{6} \\ y &= \pm\frac{1}{\sqrt{6}} \end{aligned}$$

If $y = \frac{1}{\sqrt{6}}$, then $x = -2\left(\frac{1}{\sqrt{6}}\right) = -\frac{2}{\sqrt{6}}$. If $y = -\frac{1}{\sqrt{6}}$, then $x = -2\left(-\frac{1}{\sqrt{6}}\right) = \frac{2}{\sqrt{6}}$. So the points where the tangent line has slope 1 are

$$\left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) \qquad \left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$$

65. skip

66. skip

67. (a)

$$\begin{aligned} h(x) &= f(x)g(x) \\ h'(x) &= f(x)g'(x) + f'(x)g(x) \\ h'(1) &= f(1)g'(1) + f'(1)g(1) \\ &= (3)(4) + (-2)(5) \\ &= 12 - 10 \\ &= 2 \end{aligned}$$

(b)

$$\begin{aligned} F(x) &= f(g(x)) \\ F'(x) &= f'(g(x))g'(x) \\ F'(1) &= f'(g(1))g'(1) \\ &= f'(5)g'(1) \\ &= (11)(4) \\ &= 44 \end{aligned}$$

68. skip

69.

$$\begin{aligned} f(x) &= x^2 g(x) \\ f'(x) &= x^2 g'(x) + 2x g(x) \end{aligned}$$

70.

$$\begin{aligned} f(x) &= g(x^2) \\ f'(x) &= g'(x^2) \cdot 2x \\ &= 2x g'(x^2) \end{aligned}$$

71.

$$\begin{aligned} f(x) &= [g(x)]^2 \\ f'(x) &= 2[g(x)]g'(x) \end{aligned}$$

72.

$$\begin{aligned} f(x) &= g(g(x)) \\ f'(x) &= g'(g(x))g'(x) \end{aligned}$$

73.

$$\begin{aligned} f(x) &= g(e^x) \\ f'(x) &= g'(e^x)e^x \end{aligned}$$

74.

$$\begin{aligned} f(x) &= e^{g(x)} \\ f'(x) &= e^{g(x)} g'(x) \end{aligned}$$

75. skip

76. skip

77.

$$\begin{aligned}
h(x) &= \frac{f(x)g(x)}{f(x)+g(x)} \\
h'(x) &= \frac{\Big(f(x)+g(x)\Big)\frac{d}{dx}[f(x)g(x)] - f(x)g(x)\frac{d}{dx}[f(x)+g(x)]}{[f(x)+g(x)]^2} \\
&= \frac{\Big(f(x)+g(x)\Big)\Big(f(x)g'(x)+f'(x)g(x)\Big) - f(x)g(x)\Big(f'(x)+g'(x)\Big)}{[f(x)+g(x)]^2} \\
&= \frac{[f(x)]^2g'(x)+f(x)f'(x)g(x)+f(x)g(x)g'(x)+f'(x)[g(x)]^2 - f(x)f'(x)g(x) - f(x)g(x)g}{[f(x)+g(x)]^2} \\
&= \frac{[f(x)]^2g'(x)+f'(x)[g(x)]^2}{[f(x)+g(x)]^2}
\end{aligned}$$

78.

$$\begin{aligned}
h(x) &= \sqrt{\frac{f(x)}{g(x)}} \\
&= \left(\frac{f(x)}{g(x)}\right)^{\frac{1}{2}} \\
h'(x) &= \frac{1}{2}\left(\frac{f(x)}{g(x)}\right)^{-\frac{1}{2}} \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] \\
&= \frac{1}{2}\left(\frac{g(x)}{f(x)}\right)^{\frac{1}{2}} \left(\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}\right) \\
&= \frac{g(x)f'(x)-f(x)g'(x)}{2[f(x)]^{\frac{1}{2}}[g(x)]^{\frac{3}{2}}} \\
&= \frac{g(x)f'(x)-f(x)g'(x)}{2\sqrt{f(x)}\sqrt{[g(x)]^3}}
\end{aligned}$$

79.

$$\begin{aligned}
h(x) &= f(g(\sin 4x)) \\
h'(x) &= f'(g(\sin 4x))\frac{d}{dx}[g(\sin 4x)] \\
&= f'(g(\sin 4x))g'(\sin 4x)\frac{d}{dx}(4x) \\
&= 4f'(g(\sin 4x))g'(\sin 4x)
\end{aligned}$$

80. skip

81. skip

82. (a)

$$\begin{aligned} x - 4y &= 1 \\ -4y &= -x + 1 \\ y &= \frac{1}{4}x - \frac{1}{4} \end{aligned}$$

Since parallel lines have the same slope, we want a tangent line with slope $m = \frac{1}{4}$. But the slope of the tangent line is given by

$$y' = e^x$$

so we have

$$\begin{aligned} \frac{1}{4} &= e^x \\ \ln\frac{1}{4} &= \ln e^x \\ \ln 4^{-1} &= x \\ -\ln 4 &= x \end{aligned}$$

Since $y = e^x$, we have $y = e^{\ln\frac{1}{4}} = \frac{1}{4}$, so the equation of our tangent line is

$$\begin{aligned} y - \frac{1}{4} &= \frac{1}{4}(x - (-\ln 4)) \\ y - \frac{1}{4} &= \frac{1}{4}x + \frac{1}{4}\ln 4 \\ y &= \frac{1}{4}x + \frac{1}{4}\ln 4 + \frac{1}{4} \end{aligned}$$

(b) Let the point of tangency be (x, e^x). If the tangent line goes through the origin, it goes through the point $(0, 0)$. Since we have two points on the tangent line we can compute the slope of the tangent line

$$\begin{aligned} m &= \frac{e^x - 0}{x - 0} \\ &= \frac{e^x}{x} \end{aligned}$$

But the slope of the tangent line is given by $y' = e^x$ so we have

$$\begin{aligned} \frac{e^x}{x} &= e^x \\ e^x &= xe^x \\ 0 &= xe^x - e^x \\ 0 &= e^x(x - 1) \end{aligned}$$

So either $e^x = 0$ (which is impossible) or $x - 1 = 0$. Therefore $x = 1$. If $x = 1$, we have $m = e^{(1)}$ so the equation of the tangent line is

$$\begin{aligned} y - 0 &= e(x - 0) \\ y &= ex \end{aligned}$$

83. The slope of the line tangent to $y = ax^2 + bx + c$ is given by $y' = 2ax + b$. At $x = -1$, the tangent has slope $m = 6$, so

$$2a(-1) + b = 6$$

At $x = 5$, the tangent has slope $m = -2$, so

$$2a(5) + b = -2$$

Now we have the system of equations

$$\begin{aligned} -2a + b &= 6 \\ 10a + b &= -2 \end{aligned}$$

If we subtract the two equations, we have

$$\begin{aligned} -12a &= 8 \\ a &= \frac{8}{-12} \\ &= -\frac{2}{3} \end{aligned}$$

Since $-2a + b = 6$, we have

$$\begin{aligned} -2\left(-\frac{2}{3}\right) + b &= 6 \\ \frac{4}{3} + b &= 6 \\ b &= \frac{14}{3} \end{aligned}$$

So we have $y = -\frac{2}{3}x^2 + \frac{14}{3}x + c$. To find c, we know the parabola goes through the point $(1, 4)$ so we have

$$\begin{aligned} 4 &= -\frac{2}{3}(1)^2 + \frac{14}{3}(1) + c \\ 4 &= -\frac{2}{3} + \frac{14}{3} + c \\ 4 &= 4 + c \\ 50 &= c \end{aligned}$$

Therefore our parabola is

$$y = -\frac{2}{3}x^2 + \frac{14}{3}x$$

84. skip

85. skip

86. (a)

$$\begin{aligned}
f(t) &= \sqrt{b^2 + c^2t^2} \\
&= (b^2 + c^2t^2)^{\frac{1}{2}} \\
v(t) &= f'(t) \\
&= \frac{1}{2}(b^2 + c^2t^2)^{-\frac{1}{2}} \frac{d}{dt}(b^2 + c^2t^2) \\
&= \frac{1}{2}(b^2 + c^2t^2)^{-\frac{1}{2}}(2c^2t) \\
&= c^2t(b^2 + c^2t^2)^{-\frac{1}{2}} \\
a(t) &= f''(t) \\
&= c^2t\frac{d}{dt}(b^2 + c^2t^2)^{-\frac{1}{2}} + (b^2 + c^2t^2)^{-\frac{1}{2}}\frac{d}{dt}(c^2t) \\
&= c^2t\left(-\frac{1}{2}(b^2 + c^2t^2)^{-\frac{3}{2}}\frac{d}{dt}(b^2 + c^2t^2)\right) + (b^2 + c^2t^2)^{-\frac{1}{2}}(c^2) \\
&= c^2t\left(-\frac{1}{2}(b^2 + c^2t^2)^{-\frac{3}{2}}(2c^2t)\right) + c^2(b^2 + c^2t^2)^{-\frac{1}{2}} \\
&= -c^4t^2(b^2 + c^2t^2)^{-\frac{3}{2}} + c^2(b^2 + c^2t^2)^{-\frac{1}{2}} \\
&= \frac{-c^4t^2}{(b^2 + c^2t^2)^{\frac{3}{2}}} + \frac{c^2}{(b^2 + c^2t^2)^{\frac{1}{2}}} \\
&= \frac{-c^4t^2 + c^2(b^2 + c^2t^2)}{(b^2 + c^2t^2)^{\frac{3}{2}}} \\
&= \frac{-c^4t^2 + c^2b^2 + c^4t^2}{(b^2 + c^2t^2)^{\frac{3}{2}}} \\
&= \frac{c^2b^2}{(b^2 + c^2t^2)^{\frac{3}{2}}}
\end{aligned}$$

(b) The particle moves in the positive direction when the velocity is positive. Since the velocity is

$$v(t) = c^2t(b^2 + c^2t^2)^{-\frac{1}{2}} = \frac{c^2t}{\sqrt{b^2 + c^2t^2}}$$

we can see that the numerator is always positive since $t \geq 0$ and the denominator is always positive, so $v(t)$ is always positive and the particle always moves in the positive direction.

87. (a)

$$\begin{aligned} f(t) &= t^3 - 12t + 3 \\ v(t) &= f'(t) \\ &= 3t^2 - 12 \\ a(t) &= f''(t) \\ &= 6t \end{aligned}$$

(b) The particle is moving upward when $v(t) > 0$ and downward when $v(t) < 0$. Since

$$v(t) = 3t^2 - 12 = 3(t^2 - 4) = 3(t-2)(t+2)$$

we know that

	$t < -2$	$-2 < t < 2$	$2 < t$
$t-2$	$-$	$-$	$+$
$t+2$	$-$	$+$	$+$
$(t-2)(t+2)$	$+$	$-$	$+$

So $v(t) > 0$ when $t < -2$ and $t > 2$, but we are only concerned with $t \geq 0$, so the particle is moving upward when $t > 2$. $v(t) < 0$ when $-2 < t < 2$, but we are only concerned with $t \geq 0$, so the particle is moving downward when $0 \leq t < 2$.

(c) From $t = 0$ to $t = 2$ the particle is moving downward. From $t = 2$ on the particle is moving upward, so the total distance travelled from $t = 0$ to $t = 3$ is

$$\begin{aligned} |f(0) - f(2)| + |f(2) - f(3)| &= |((0)^3 - 12(0) + 3) - ((2)^3 - 12(2) + 3)| \\ &\quad +|((2)^3 - 12(2) + 3) - ((3)^3 - 12(3) + 3)| \\ &= |3 - (-13)| + |-13 - (-6)| \\ &= |16| + |-7| \\ &= 16 + 7 \\ &= 23 \end{aligned}$$

88.-99. skip

100.

$$\lim_{x \to 1} \frac{x^{17} - 1}{x - 1}$$

represents the derivative of the function $f(x) = x^{17}$ at the point $a = 1$. Since

$$f'(x) = 17x^{16}$$

we have $f'(1) = 17(1)^{16} = 17$.

101.

$$\lim_{h \to 0} \frac{\sqrt[4]{16 + h} - 2}{h}$$

represents the derivative of the function $f(x) = \sqrt[4]{x}$ at the point $a = 16$. Since

$$f'(x) = \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{4\sqrt[4]{x^3}}$$

we have $f'(16) = \dfrac{1}{4\sqrt[4]{16^3}} = \dfrac{1}{32}$.

102.

$$\lim_{\theta \to \frac{\pi}{3}} \frac{\cos\theta - \frac{1}{2}}{\theta - \frac{\pi}{3}}$$

represents the derivative of the function $f(\theta) = \cos\theta$ at the point $a = \dfrac{\pi}{3}$. Since

$$f'(\theta) = -\sin\theta$$

we have $f'\left(\dfrac{\pi}{3}\right) = -\sin\left(\dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$.

103. skip

104. Suppose f is a differentiable function such that $f(g(x)) = x$ and $f'(x) = 1+[f(x)]^2$. Show that $g'(x) = \dfrac{1}{1+x^2}$.

First notice that

$$\begin{aligned} \frac{d}{dx}\Big[f(g(x))\Big] &= \frac{d}{dx}[x] \\ f'(g(x))g'(x) &= 1 \\ g'(x) &= \frac{1}{f'(g(x))} \end{aligned}$$

However, since $f'(x) = 1 + [f(x)]^2$, we have

$$f'(g(x)) = 1 + [f(g(x))]^2$$

But $f(g(x)) = x$ so this becomes

$$f'(g(x)) = 1 + x^2$$

So we have our result

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{1+x^2}$$

105. Find $f'(x)$ if it is known that

$$\frac{d}{dx}[f(2x)] = x^2$$

First, notice that

$$\frac{d}{dx}[f(2x)] = f'(2x)\frac{d}{dx}[2x] = f'(2x) \cdot 2$$

So we have

$$f'(2x) = \frac{x^2}{2}$$

Let $t = 2x$. Then $x = \dfrac{t}{2}$ and we have

$$\begin{aligned} f'(t) &= \frac{(\frac{t}{2})^2}{2} \\ &= \frac{\frac{t^2}{4}}{2} \\ &= \frac{t^2}{8} \end{aligned}$$

So

$$f'(x) = \frac{x^2}{8}$$