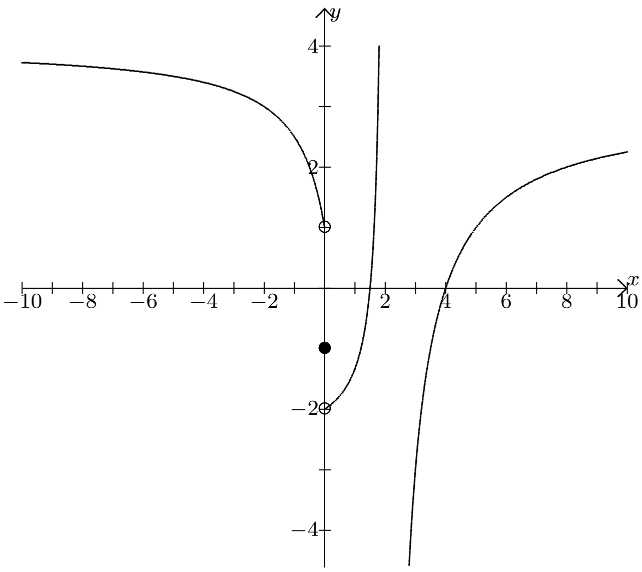
1. (a) i. 3
   ii. 0
   iii. Does Not Exist (the left and right limits are not equal)
   iv. 2
   v. Does Not Exist (or ∞)
   vi. Does Not Exist (or $-\infty$)
   vii. 4
   viii. -1

   (b) $y = 4$ and $y = -1$

   (c) $x = 0$ and $x = 2$

   (d) $x = -3$ (jump discontinuity), $x = 0$ and $x = 2$ (infinite discontinuity), and $x = 4$ (removable discontinuity)

2. Just one example of a possible graph:



3.

$$\begin{aligned}\lim_{x\to 1} e^{x^3-x} &= e^{1^3-1}\\ &= e^0\\ &= 1\end{aligned}$$

4.

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-9}{x^2+2x-3} &= \frac{(3)^2-9}{(3)^2+2(3)-3}\\ &= \frac{0}{12}\\ &= 0\end{aligned}$$

5.

$$\begin{aligned}\lim_{x\to -3}\frac{x^2-9}{x^2+2x-3} &= \lim_{x\to -3}\frac{(x-3)(x+3)}{(x-1)(x+3)}\\ &= \lim_{x\to -3}\frac{x-3}{x-1}\\ &= \frac{(-3)-3}{(-3)-1}\\ &= \frac{-6}{-4}\\ &= \frac{3}{2}\end{aligned}$$

6.

$$\lim_{x\to 1^+}\frac{x^2-9}{x^2+2x-3} = \frac{-8}{0}\to -\infty$$

7.

$$\begin{aligned}\lim_{h\to 0}\frac{(h-1)^3+1}{h} &= \lim_{h\to -0}\frac{h^3-3h^2+3h-1+1}{h}\\ &= \lim_{h\to 0}\frac{h^3-3h^2+3h}{h}\\ &= \lim_{h\to 0}(h^2-3h+3)\\ &= (0)^2-3(0)+3\\ &= 3\end{aligned}$$

8. skip

9.

$$\lim_{r\to 9}\frac{\sqrt{r}}{(r-9)^4} = \frac{3}{0}\to\infty$$

10.

$$\begin{aligned}\lim_{v\to 4^+}\frac{4-v}{|4-v|} &= \lim_{v\to 4^+}\frac{4-v}{-(4-v)}\\ &= \lim_{v\to 4^+}-1\\ &= -1\end{aligned}$$

11.

$$\begin{aligned}
\lim_{s\to 16}\frac{4-\sqrt{s}}{s-16} &= \lim_{s\to 16}\frac{(4-\sqrt{s})(4+\sqrt{s})}{(s-16)(4+\sqrt{s})}\\
&= \lim_{s\to 16}\frac{16-s}{(s-16)(4+\sqrt{s})}\\
&= \lim_{s\to 16}\frac{-(s-16)}{(s-16)(4+\sqrt{s})}\\
&= \lim_{s\to 16}\frac{-1}{4+\sqrt{s}}\\
&= \frac{-1}{4+\sqrt{16}}\\
&= -\frac{1}{8}
\end{aligned}$$

12.

$$\begin{aligned}
\lim_{v\to 2}\frac{v^2+2v-8}{v^4-16} &= \lim_{v\to 2}\frac{(v-2)(v+4)}{(v^2+4)(v^2-4)}\\
&= \lim_{v\to 2}\frac{(v-2)(v+4)}{(v^2+4)(v-2)(v+2)}\\
&= \lim_{v\to 2}\frac{v+4}{(v^2+4)(v+2)}\\
&= \frac{(2)+4}{((2)^2+4)((2)+2}\\
&= \frac{6}{(8)(4)}\\
&= \frac{6}{32}\\
&= \frac{3}{16}
\end{aligned}$$

13.

$$\begin{aligned}
\lim_{x\to 8^-}\frac{|x-8|}{x-8} &= \lim_{x\to 8^-}\frac{-(x-8)}{x-8}\\
&= \lim_{x\to 8^-}-1\\
&= -1
\end{aligned}$$

14. skip

15.

$$\begin{aligned}
\lim_{x\to 0}\frac{1-\sqrt{1-x^2}}{x} &= \lim_{x\to 0}\frac{(1-\sqrt{1-x^2})(1+\sqrt{1-x^2})}{x(1+\sqrt{1-x^2})} \\
&= \lim_{x\to 0}\frac{1-(1-x^2)}{x(1+\sqrt{1-x^2})} \\
&= \lim_{x\to 0}\frac{x^2}{x(1+\sqrt{1-x^2})} \\
&= \lim_{x\to 0}\frac{x}{1+\sqrt{1-x^2}} \\
&= \frac{0}{1+\sqrt{1-(0)^2}} \\
&= \frac{0}{2} \\
&= 0
\end{aligned}$$

16.

$$\begin{aligned}
\lim_{x\to 2}\frac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x} &= \lim_{x\to 2}\frac{(\sqrt{x+2}-\sqrt{2x})(\sqrt{x+2}+\sqrt{2x})}{(x^2-2x)(\sqrt{x+2}+\sqrt{2x})} \\
&= \lim_{x\to 2}\frac{(x+2)-(2x)}{x(x-2)(\sqrt{x+2}+\sqrt{2x})} \\
&= \lim_{x\to 2}\frac{2-x}{x(x-2)(\sqrt{x+2}+\sqrt{2x})} \\
&= \lim_{x\to 2}\frac{-(x-2)}{x(x-2)(\sqrt{x+2}+\sqrt{2x})} \\
&= \lim_{x\to 2}\frac{-1}{x(\sqrt{x+2}+\sqrt{2x})} \\
&= \frac{-1}{(2)(\sqrt{(2)+2}+\sqrt{2(2)})} \\
&= \frac{-1}{2(2+2)} \\
&= -\frac{1}{8}
\end{aligned}$$

17.

$$\begin{aligned}
\lim_{x\to\infty}\frac{1+2x-x^2}{1-x+2x^2} &= \lim_{x\to\infty}\frac{\frac{1+2x-x^2}{x^2}}{\frac{1-x+2x^2}{x^2}} \\
&= \lim_{x\to\infty}\frac{\frac{1}{x^2}+\frac{2}{x}-1}{\frac{1}{x^2}-\frac{1}{x}+2} \\
&= \frac{0+0-1}{0-0+2} \\
&= -\frac{1}{2}
\end{aligned}$$

18.

$$\begin{aligned}
\lim_{x\to-\infty}\frac{5x^3-x^2+2}{2x^3+x-3} &= \lim_{x\to-\infty}\frac{\frac{5x^3-x^2+2}{x^3}}{\frac{2x^3+x-3}{x^3}} \\
&= \lim_{x\to-\infty}\frac{5-\frac{1}{x}+\frac{2}{x^3}}{2+\frac{1}{x^2}-\frac{3}{x^3}} \\
&= \frac{5-0+0}{2+0-0} \\
&= \frac{5}{2}
\end{aligned}$$

19.

$$\begin{aligned}
\lim_{x\to\infty}\frac{\sqrt{x^2-9}}{2x-6} &= \lim_{x\to\infty}\frac{\frac{\sqrt{x^2-9}}{x}}{\frac{2x-6}{x}} \\
&= \lim_{x\to\infty}\frac{\frac{\sqrt{x^2-9}}{\sqrt{x^2}}}{\frac{2x-6}{x}} \\
&= \lim_{x\to\infty}\frac{\sqrt{\frac{x^2-9}{x^2}}}{\frac{2x-6}{x}} \\
&= \lim_{x\to\infty}\frac{\sqrt{1-\frac{9}{x^2}}}{2-\frac{6}{x}} \\
&= \frac{\sqrt{1-0}}{2-0} \\
&= \frac{1}{2}
\end{aligned}$$

20. skip

21. skip

22. skip

23. skip

24. skip

25. Notice that

$$\lim_{x\to 1}(2x-1) = 2(1) - 1 = 1$$

and

$$\lim_{x\to 1} x^2 = (1)^2 = 1$$

Since $2x - 1 \le f(x) \le x^2$, by the Squeeze Theorem, $\lim_{x\to 1} f(x) = 1$

26. skip

27. skip

28. skip

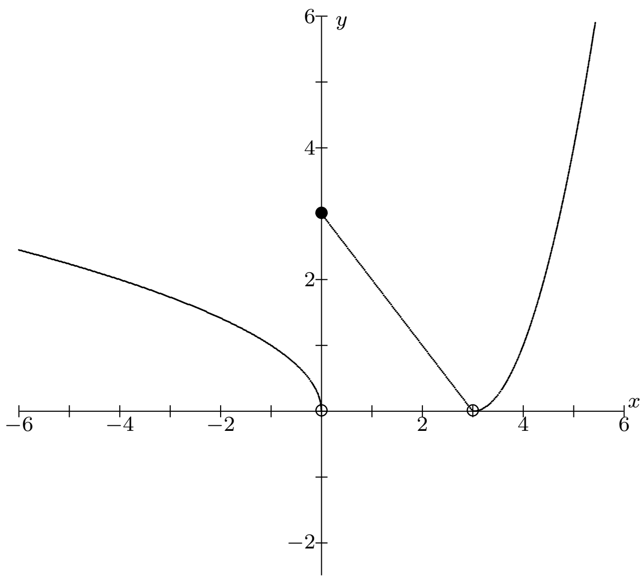
29. skip

30. skip

31. (a)
    i. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (3-x) = 3-(0) = 3$
    ii. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt{-x} = \sqrt{-(0)} = 0$
    iii. $\lim_{x\to 0} f(x)$ does not exist because the left and right limits are not the same.
    iv. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (3-x) = 3-(3) = 0$
    v. $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x-3)^2 = ((3)-3)^2 = 0$
    vi. $\lim_{x\to 3} f(x) = 0$

    (b) f is discontinuous when $x = 0$ (because the limit does not exist) and when $x = 3$ (because the function is not defined there)

    (c) .



32. (a) When $x = 2$ we have

$$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} (2x - x^2) = 2(2) - (2)^2 = 0$$

and

$$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} (2 - x) = 2 - (2) = 0$$

So $\lim_{x\to 2} g(x) = 0 = g(2)$, and g is continuous at 2 (and hence continuous from the left and right).
When $x = 3$ we have

$$\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-} (2 - x) = 2 - (3) = -1$$

and

$$\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+} (x - 4) = (3) - 4 = -1$$

So $\lim_{x\to 3} g(x) = -1 = g(3)$, and g is continuous at 3 (and hence continuous from the left and right).
When $x = 4$ we have

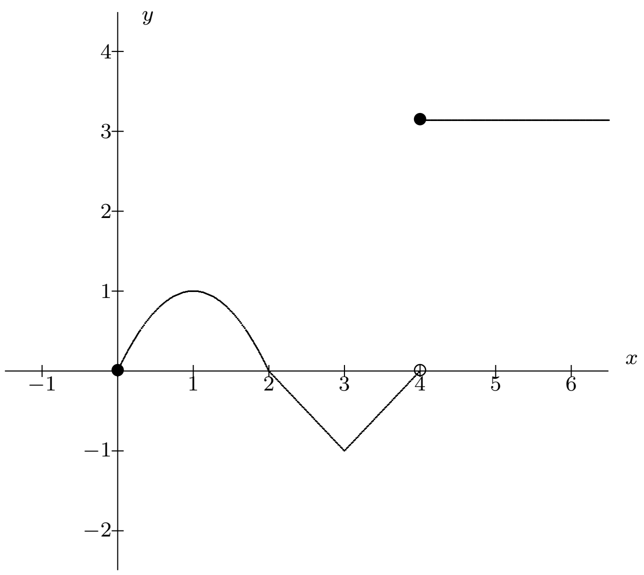
$$\lim_{x\to 4^-} g(x) = \lim_{x\to 4^-} (x - 4) = (4) - 4 = 0$$

and

$$\lim_{x\to 4^+} g(x) = \lim_{x\to 4^+} \pi = \pi$$

So $\lim_{x\to 4} g(x)$ does not exist, and g is not continuous at 4. Since $\lim_{x\to 4^-} g(x) \neq f(4)$ and $\lim_{x\to 4^+} g(x) = g(4)$, $g(x)$ is continuous from the right but not the left at $x = 4$.

(b) .



33. skip

34. skip

35. $f(x) = 2x^3 + x^2 + 2$
Notice
$$f(-2) = 2(-2)^3 + (-2)^2 + 2 = -10$$
and
$$f(-1) = 2(-1)^3 + (-1)^2 + 2 = 1$$
By Intermediate Value Theorem, there exists a number c, $-1 \leq c \leq 2$ so that $f(c) = 0$.

36. $f(x) = e^{-x^2} - x$
Notice
$$f(0) = e^{(-0^2)} - (0) = 1$$
and
$$f(1) = e^{-(1)^2} - (1) \approx -0.642$$
By Intermediate Value Theorem, there exists a number c, $0 \leq c \leq 1$ so that $f(c) = 0$.

37. (a) The derivative of $y = f(x)$ when $x = 2$ is the slope of the line tangent to y at $(2, 1)$.
$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h\to 0} \frac{[9 - 2(2+h)^2] - [9 - 2(2)^2]}{h} \\ &= \lim_{h\to 0} \frac{9 - 8 - 8h - 2h^2 - 1}{h} \\ &= \lim_{h\to 0} \frac{-8h - 2h^2}{h} \\ &= \lim_{h\to 0}(-8 - 2h) \\ &= -8 - 2(0) \\ &= -8 \end{aligned}$$

(b) The line tangent to $y = f(x)$ at $(2, 1)$ has slope -8 so its equation is
$$\begin{aligned} y - 1 &= (-8)(x - 2) \\ y - 1 &= -8x + 16 \\ y &= -8x + 17 \end{aligned}$$

38. The slope of the line tangent to $y = f(x)$ at $x = a$ is $f'(a)$. For $x = a$, the slope of the

tangent line is

$$\begin{aligned}
f'(a) &= \lim_{h\to a}\frac{f(a+h)-f(a)}{h}\\
&= \lim_{h\to 0}\frac{\left[\frac{2}{1-3(a+h)}\right]-\left[\frac{2}{1-3(a)}\right]}{h}\\
&= \lim_{h\to 0}\frac{\frac{2}{1-3a-3h}-\frac{2}{1-3a}}{h}\\
&= \lim_{h\to 0}\left(\frac{2(1-3a)}{(1-3a-3h)(1-3a)}-\frac{2(1-3a-3h)}{(1-3a)(1-3a-3h)}\right)\frac{1}{h}\\
&= \lim_{h\to 0}\frac{(2-6a)-(2-6a-6h)}{h(1-3a)(1-3a-3h)}\\
&= \lim_{h\to 0}\frac{6h}{h(1-3a)(1-3a-3h)}\\
&= \lim_{h\to 0}\frac{6}{(1-3a)(1-3a-3h)}\\
&= \frac{6}{(1-3a)(1-3a-3(0))}\\
&= \frac{6}{(1-3a)^2}
\end{aligned}$$

When $x=0$, the slope of the tangent line is $f'(0)=\dfrac{6}{(1-3(0))^2}=6$. The point of tangency is $(0,f(0))$ and $f(0)=\dfrac{2}{1-3(0)}=2$, so the equation of the line tangent to $y=f(x)$ at $(0,2)$ is

$$\begin{aligned}
y-2 &= 6(x-0)\\
y-2 &= 6x\\
y &= 6x+2
\end{aligned}$$

When $x=-1$, the slope of the tangent line is $f'(-1)=\dfrac{6}{(1-3(-1))^2}=\dfrac{3}{8}$. The point of tangency is $(-1,f(-1))$ and $f(-1)=\dfrac{2}{1-3(-1)}=\dfrac{1}{2}$, so the equation of the line tangent to $y=f(x)$ at $(-1,\frac{1}{2})$ is

$$\begin{aligned}
y-\frac{1}{2} &= \frac{3}{8}(x-(-1))\\
y-\frac{1}{2} &= \frac{3}{8}x+\frac{3}{8}\\
y &= \frac{3}{8}x+\frac{7}{8}
\end{aligned}$$

39. (a) Average velocity is

$$\frac{\text{displacement}}{\text{time}}$$

i.

$$\begin{aligned}\frac{s(3)-s(1)}{3-1} &= \frac{(1+2(3)+\frac{(3)^2}{4})-(1+2(1)+\frac{(1)^2}{4})}{3-1}\\ &= \frac{\frac{37}{4}-\frac{13}{4}}{2}\\ &= 3 \quad \text{m/s}\end{aligned}$$

ii.

$$\begin{aligned}\frac{s(2)-s(1)}{2-1} &= \frac{(1+2(2)+\frac{(2)^2}{4})-(1+2(1)+\frac{(1)^2}{4})}{2-1}\\ &= \frac{6-\frac{13}{4}}{1}\\ &= 2.75 \quad \text{m/s}\end{aligned}$$

iii.

$$\begin{aligned}\frac{s(1.5)-s(1)}{1.5-1} &= \frac{(1+2(1.5)+\frac{(1.5)^2}{4})-(1+2(1)+\frac{(1)^2}{4})}{1.5-1}\\ &= \frac{\frac{73}{16}-\frac{13}{4}}{0.5}\\ &= 2.625 \quad \text{m/s}\end{aligned}$$

iv.

$$\begin{aligned}\frac{s(1.1)-s(1)}{1.1-1} &= \frac{(1+2(1.1)+\frac{(1.1)^2}{4})-(1+2(1)+\frac{(1)^2}{4})}{1.1-1}\\ &= \frac{\frac{1401}{400}-\frac{13}{4}}{0.1}\\ &= 2.525 \quad \text{m/s}\end{aligned}$$

(b) The instantaneous velocity of an object with position function $s = f(t)$ when $t = 1$

is given by

$$\begin{aligned}
f'(1) &= \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}\\
&= \lim_{h\to 0}\frac{[1+2(1+h)+\frac{(1+h)^2}{4}]-[1+2(1)+\frac{(1)^2}{4}]}{h}\\
&= \lim_{h\to 0}\frac{1+2+2h+\frac{1+2h+h^2}{4}-1-2-\frac{1}{4}}{h}\\
&= \lim_{h\to 0}\frac{\frac{5}{2}h+\frac{1}{4}h^2}{h}\\
&= \lim_{h\to 0}\left(\frac{5}{2}+\frac{1}{4}h\right)\\
&= \frac{5}{2}+\frac{1}{4}(0)\\
&= \frac{5}{2} \quad \text{m/s}
\end{aligned}$$

40. skip

41. (a)

$$\begin{aligned}
f'(2) &= \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}\\
&= \lim_{h\to 0}\frac{[(2+h)^3-2(2+h)]-[2^3-2(2)]}{h}\\
&= \lim_{h\to 0}\frac{8+12h+6h^2+h^3-4-2h-8+4}{h}\\
&= \lim_{h\to 0}\frac{10h+6h^2+h^3}{h}\\
&= \lim_{h\to 0}(10+6h+h^2)\\
&= 10+6(0)+(0)^2\\
&= 10
\end{aligned}$$

(b) The line tangent to $y=f(x)$ at $(2,4)$ has slope $f'(2)$ so we have

$$\begin{aligned}
y-4 &= 10(x-2)\\
y-4 &= 10x-20\\
y &= 10x-16
\end{aligned}$$

(c) skip

42. Recall that

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

so if

$$f'(a) = \lim_{h\to 0} \frac{(2+h)^6 - 64}{h}$$

$f(x) = x^6$ and $a = 2$ is one possibility. Another possibility is $f(x) = (x+2)^6$ and $a = 0$.

43. skip

44. graph

45. graph

46. graph

47. (a)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{3 - 5(x+h)} - \sqrt{3-5x}}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{3-5x-5h} - \sqrt{3-5x})(\sqrt{3-5x-5h} + \sqrt{3-5x})}{h(\sqrt{3-5x-5h} + \sqrt{3-5x})} \\
&= \lim_{h\to 0} \frac{(3-5x-5h) - (3-5x)}{h(\sqrt{3-5x-5h} + \sqrt{3-5x})} \\
&= \lim_{h\to 0} \frac{-5h}{h(\sqrt{3-5x-5h} + \sqrt{3-5x})} \\
&= \lim_{h\to 0} \frac{-5}{\sqrt{3-5x-5h} + \sqrt{3-5x}} \\
&= \frac{-5}{\sqrt{3-5x-5(0)} + \sqrt{3-5x}} \\
&= \frac{-5}{\sqrt{3-5x} + \sqrt{3-5x}} \\
&= -\frac{5}{2\sqrt{3-5x}}
\end{aligned}$$

(b) Domain of $f(x) = \sqrt{3-5x}$ is

$$\begin{aligned}
3 - 5x &\geq 0 \\
3 &\geq 5x \\
\frac{3}{5} &\geq x
\end{aligned}$$

Domain of $f'(x) = -\dfrac{5}{2\sqrt{3-5x}}$ is

$$\begin{aligned} 3-5x &> 0 \\ 3 &> 5x \\ \frac{3}{5} &> x \end{aligned}$$

(c) skip

48. (a) The horizontal asymptote(s) are found by

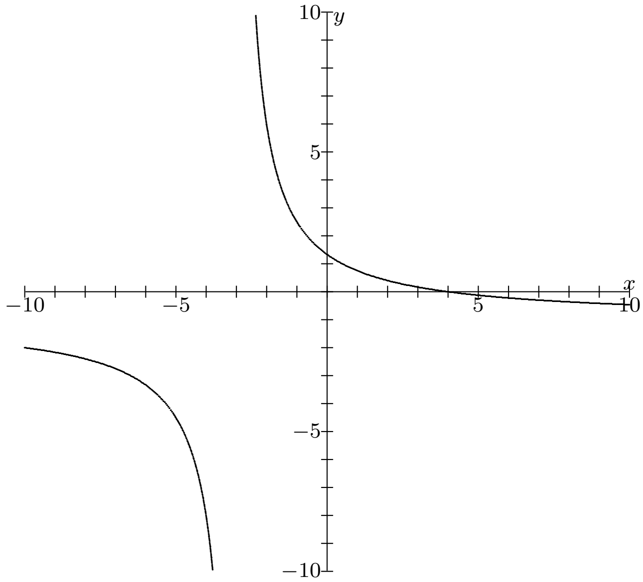
$$\lim_{x\to\pm\infty}\frac{4-x}{3+x} = \lim_{x\to\pm\infty}\frac{\frac{4-x}{x}}{\frac{3+x}{x}} = \lim_{x\to\infty}\frac{\frac{4}{x}-1}{\frac{3}{x}+1} = \frac{0-1}{0+1} = -1$$

The vertical asymptotes are places where the function increases or decreases without bound (where the denominator is zero) so we have a vertical asymptote at $x=-3$ Notice that

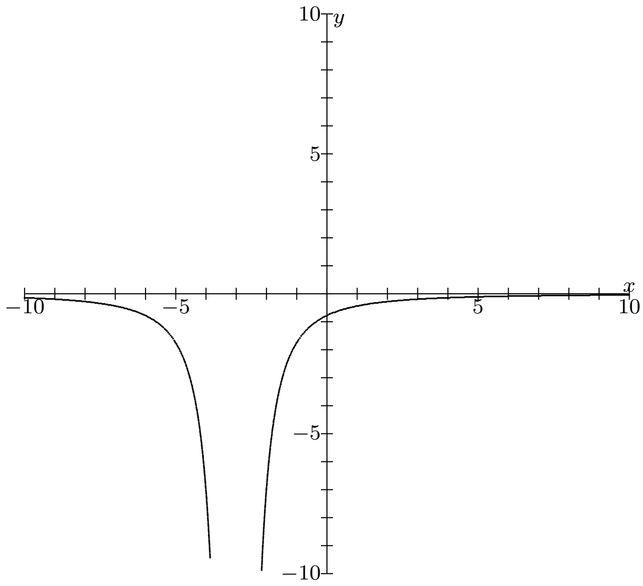
$$\lim_{x\to -3^-}\frac{4-x}{3+x} = -\infty \qquad (4-x>0 \text{ and } 3+x<0)$$

and

$$\lim_{x\to -3^+}\frac{4-x}{3+x} = \infty \qquad (4-x>0 \text{ and } 3+x>0)$$

So the graph looks like 

(b) The derivative will look like



(c)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\frac{4-(x+h)}{3+(x+h)}-\frac{4-x}{3+x}}{h}\\
&= \lim_{h\to 0}\frac{\frac{4-x-h}{3+x+h}-\frac{4-x}{3+x}}{h}\\
&= \lim_{h\to 0}\left(\frac{(4-x-h)(3+x)}{(3+x+h)(3+x)}-\frac{(4-x)(3+x+h)}{(3+x)(3+x+h)}\right)\frac{1}{h}\\
&= \lim_{h\to 0}\frac{(12-3x-3h+4x-x^2-xh)-(12+4x+4h-3x-x^2-xh)}{h(3+x)(3+x+h)}\\
&= \lim_{h\to 0}\frac{-7h}{h(3+x)(3+x+h)}\\
&= \lim_{h\to 0}\frac{-7}{(3+x)(3+x+h)}\\
&= \frac{-7}{(3+x)(3+x+(0))}\\
&= \frac{-7}{(3+x)(3+x)}\\
&= -\frac{7}{(3+x)^2}
\end{aligned}$$

(d) skip

49. The function is not differentiable at $x = -4$ and $x = 2$ (because it is discontinuous there), at $x = 5$ (it has a vertical tangent there), and $x = -1$ (it has a sharp corner there)