Directions: Solve the following problem using calculus. You can use any source (books, internet or people) provided that you properly cite it. Your solution should be in the form of a type-written report with proper spelling, grammar and punctuation. You can be as creative as you like, but it must not simply be a list of formulas, rather a communication in writing of mathematical ideas using words.

The year is 2020. Hockey fans throughout Manitoba are following the fortunes of the Winnipeg Gliders, the city's entry in the new, co-ed Intercontinental Hockey League. But things are not going well for the Gliders; the team is mired in last place, game attendance and revenues are down, and the owners of the Gliders are threatening to move the team to Buenos Aires, which desperately wants an ICHL franchise.

One morning you are sitting in the office of Math Iz Us (the small consulting firm that you and your two partners have recently opened after having difficulty finding satisfactory summer jobs) when the phone rings. On the line is Jacques Schtrop, the Glider's coach.

"I think that we may have the solution to our team's problems!" he tells you. "We've just signed the young European superstar Tina Salami, and she'll be joining the team next week. According to our scouts she has a slapshot that's been clocked at 135 miles per hour. Even if she's far from the goal when she shoots, the puck travels so fast that the goalie doesn't have time to react before it's in the net. And if the poor guy happens to be in the way of the puck, it knocks him right into the net and follows him in. We've never seen anything like it!"

"That's wonderful news!" you reply. "But how can I help you?"

"Tina has one little problem," continues Jacques. "She lacks accuracy. When she shoots you never know what direction the puck will go, so most of her shots aren't on goal. We've been working with her, and she's better than she used to be, but it would really help if she was subtending the largest possible angle to the goal-mouth when she shoots. That way the problem cause by her inaccuracy would be minimized. The trouble is, we don't know where she should be for that to happen."

"That's kind of vague," you say. "When you say 'largest,' what are you comparing the angle to?"

"Oh yes, I forgot to tell you about Tina's other problem," says Jacques. "She doesn't maneuver well on skates. She can skate very fast, but she can't change direction easily. So what happens is that she gets the puck at her own end of the rink and then she skates straight down the ice, along a path perpendicular to the goal-mouth, until she shoots. When she's far from the opposition goal she's subtending a small angle, and if she gets too close to the extension of the goal-mouth she's shooting from a very sharp angle, as we say in the trade.

Somewhere along that path the angle that she subtends is a maximum, and we don't know where it is. **We need that information!**"

"Of course, if she were skating directly towards the goal-mouth, then the closer she is the bigger the angle she will subtend, so ideally she should skate right to the goal-mouth before shooting," you observe.

"Sure, I know that," says Jacques, "but if the path that she skates along doesn't intersect the goal-mouth then the answer's not so obvious. It seems like a mathematical problem to me, and that's why I've called you."

"It sounds like an optimization problem," you say, "but how far from the extension of the goal-mouth she should be for the angle subtending the goal-mouth to be maximum surely must depend on the perpendicular distance of the skating path from the nearer goal-post. Look, Mr. Schtrop, let me talk it over with my partners and I'll get back to you."

"OK, but before you go, let me tell you what we would really like. We would like to get our icemaker to trace a faint green curve in the ice that would show Tina where to shoot. In other words, no matter what path she takes down the ice perpendicular to the goal-mouth, she should know that when she crosses the green line, she's at the best angle for shooting for the particular path that she is on. What we need you to do is tell our icemaker how to make that curve."

"A 'Salami curve', so to speak," you say.

"Yeah, I guess you could call it that," says Schtrop.

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When you discuss the problem with your partners, you realize that a knowledge of the Salami curve could be useful in sports other than hockey. Any sport played on a rectangular "field" requiring a player to shoot or toss a puck or ball into a goal would have the same feature. So, you decide to do a general analysis of the situation, in hopes of marketing the Salami curve to other teams besides the Gliders. Even your nephew who plays 10-year-old soccer for his community club might benefit!

Soon you realize that there are three relevant constants, namely the width of the playing surface, the length of the playing surface between the extensions of the goal-mouths, and the width of the goal-mouth. You assign letters to denote these. You also realize you must consider each "path" perpendicular to the extension separately. Each path will contain an "optimal" point, and by stringing all these points together you'll get the Salami curve. Paths intersecting the goal-mouth are easy. When you concentrate on a fixed path not intersecting the goal-mouth, you are able to express the angle subtended as the difference of two inverse trig functions. Standard optimization techniques allow you to find the “Salami point” on that particular path, and combining them gives you the Salami curve. You write down the equation of the Salami curve using the three letters assigned above as parameters (in fact, it “comes in pieces”), and draw a very accurate, neat, well-labeled diagram that is sufficiently clear for the icemaker to use as a blueprint for making the curve. Then you send the whole thing, together with an explanation of the mathematical derivation of the equation of the Salami curve, to Jacques Schtrop. He is delighted, and pays you a very large fee. You use the fee for a Hawaiian vacation, and pay no more attention to the Gliders for the rest of the season.

Epilogue With the Salami curve available for all home games, Tina became more effective than ever. She scored 174 goals by the end of the regular season and the Gliders soared to second place overall. Fans attended in droves, profits went up, and the Gliders were Clinton Cup finalists, eventually losing to the Brisbane Emus on July 28 in the last game of a best of 15 series. Nonetheless, the Gliders did not stay in Winnipeg; Buenos Aires spent 150 million dollars constructing a new arena with 60,000 upholstered seats and an indoor roller coaster, and the owners moved the Gliders there in search of a larger profit. Winnipeg was forced to abandon its own tentative plans for a new arena, and spent the money thus saved on roads, sewers, libraries, and less severe tax increases.

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Produce the package of materials that Math Iz Us mailed to Jacques Schtrop. Remember, in order to earn the full fee for the job, your work must be mathematically correct and complete, and explained clearly. Furthermore, the icemaker must be able to understand where to trace the Salami curve.

Extra credit points: Those of you who are hockey buffs and know the actual dimensions of a standard size NHL rink can determine whether the Salami curve crosses the blue line. If so, how far from the boards does this happen?