Product Rule

1. If $f(x) = x^3 \sin x$, find $f'(x)$.

Solution: The product rule states that the derivative of a product of two functions is the first function times the derivative of the second function plus the second function times the derivative of the first function.

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

We know that

$$\frac{d}{dx}(x^3) = 3x^2$$

and

$$\frac{d}{dx}(\sin x) = \cos x$$

Applying the Product Rule, we have:

$$f'(x) = 3x^2 \sin x + x^3 \cos x$$

Limits of Trig Functions

2. Find the limit: $\lim_{x\to 0} \frac{\sin 3x}{x}$

Solution: In finding the limit of any trig function, a few things to keep in mind are,

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

Using the above information lets find the limit of $\frac{\sin 3x}{x}$ as x goes to 0.
Since it is impossible to use direct substitution to find the limit of the above function, we must use another technique in solving this problem, if we multiply both numerator and denominator by 3, we have

$$\frac{\sin 3x}{x} = \frac{\sin 3x}{x} \cdot \frac{3}{3} = \frac{3 \sin 3x}{3x}$$

To proceed solving this problem we need to clearly understand that as x approaches zero, $3x$ should also be approaching to zero. Keeping that in mind, we can use

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

to solve

$$\lim_{x \to 0} \frac{3 \sin 3x}{3x}$$

To sum up,

$$\begin{aligned}
\lim_{x \to 0} \frac{\sin 3x}{x} &= \lim_{x \to 0} \frac{3 \sin 3x}{3x} \\
&= 3 \lim_{x \to 0} \frac{\sin 3x}{3x} \\
&= 3 \lim_{3x \to 0} \frac{\sin 3x}{3x} \\
&= 3(1) \\
&= 3
\end{aligned}$$

Chain Rule

3. Differentiate $y = (3x + 1)^2$

Solution: The outer layer is "the square" and the inner layer is $(3x+1)$. Differentiate "the square" first, leaving $(3x + 1)$ unchanged. Then differentiate $(3x + 1)$. So,

$$y' = 2(3x + 1)^{2-1} \frac{d}{dx}(3x + 1)$$

First the square goes out in front of the $(3x + 1)$ then the $(3x + 1)$ is taken to the 1st power. Then you differentiate $(3x + 1)$ which turns to just 3.

$$y' = 2(3x + 1)(3)$$

After all of that you multiply the equation out and come out to

$$y' = 6(3x + 1)$$

4. Find y' if $y = \sin(2x - 1)$.

Solution:

$$\begin{aligned}
\frac{d}{dx}[\sin(2x - 1)] &= \cos(2x - 1) \frac{d}{dx}[2x - 1] \\
&= 2 \cos(2x - 1)
\end{aligned}$$

Implicit Differentiation

5. Find y'' if $x^6 + y^6 = 3$.

Solution: By using implicit differentiation with respect to x, we get:

$$6x^5 + 6y^5 \frac{dy}{dx} = 0$$

Solve for y':

$$\begin{aligned} 6y^5 \frac{dy}{dx} &= -6x^5 \\ \frac{dy}{dx} &= -\frac{x^5}{y^5} \end{aligned}$$

To find y'', we differentiate using the Quotient Rule:

$$\begin{aligned} y'' &= \frac{d}{dx}\left(-\frac{x^5}{y^5}\right) \\ &= -\frac{y^5 \frac{d}{dx}(x^5) - x^5 \frac{d}{dx}(y^5)}{(y^5)^2} \\ &= -\frac{5x^4y^5 - 5x^5y^4y'}{y^{10}} \end{aligned}$$

Now substitute y' or $\frac{dy}{dx}$ into the equation:

$$\begin{aligned} y'' &= -\frac{5x^4y^5 - 5x^5y^4\left(-\frac{x^5}{y^5}\right)}{y^{10}} \\ &= -\frac{5x^4y^5 + \frac{5x^{10}}{y}}{y^{10}} \\ &= -\frac{5x^4y^5 + \frac{5x^{10}}{y}}{y^{10}} \cdot \frac{y}{y} \\ &= -\frac{5x^4y^6 + 5x^{10}}{y^{11}} \\ &= -\frac{5x^4(y^6 + x^6)}{y^{11}} \\ &= -\frac{5x^4(3)}{y^{11}} \\ &= -\frac{15x^4}{y^{11}} \end{aligned}$$

6. Find y' if $x^2 + 3y^2 = 5x$

Solution:

- First we want to differentiate each side with respect to x:

$$\frac{d}{dx}(x^2 + 3y^2) = \frac{d}{dx}(5x)$$

- When we do this we obtain:

$$2x + 6yy' = 5$$

(Remember whenever we differentiate a y in our equation we have to add a y' because we are differentiating with respect to x.)

- Next, solve the equation for y'. First subtract $2x$ from each side:

$$2x + 6yy' - 2x = 5 - 2x$$

- After canceling out $2x$ on the left side of our equation we obtain

$$6yy' = 5 - 2x$$

Now divide each side by $6y$:

$$\frac{6yy'}{6y} = \frac{5 - 2x}{6y}$$

- After canceling out $6y$ on the left side of our equation we obtain

$$y' = \frac{5 - 2x}{6y}$$

7. Find y' if $y^2 = x \arccos x - (1 - x^2)^{1/2}$

Solution: In order to go through implicit differentiation we must take the derivative of both sides of the equation and then solve for $\frac{dy}{dx}$. We start by taking the derivative of y^2 which equals $2y\frac{dy}{dx}$. Then we can start on the other side by taking the derivative of everything before the subtraction. You will have to use the product rule to differentiate the problem. We can do this by taking x times the derivative of $\arccos x$, and adding that to $\arccos x$ times the derivative of x. Then we take the derivative of the last part from the original equation by taking the $\frac{1}{2}$ and putting it in front and subtracting one from $\frac{1}{2}$ for the power. We multiply that by the derivative of the middle, which is $2x$. This gives you

$$\begin{aligned} 2y\frac{dy}{dx} &= x\frac{d}{dx}(\arccos x) + \frac{d}{dx}(x)\arccos x - \frac{1}{2}(1-x^2)^{-1/2}(-2x) \\ 2y\frac{dy}{dx} &= x\left(\frac{-1}{(1-x^2)^{1/2}}\right) + \arccos x + \frac{x}{(1-x^2)^{1/2}} \end{aligned}$$

Eventually the first part of the equation and the last part of the equation end up canceling out. This leaves us with

$$2y\frac{dy}{dx} = \arccos x$$

We now need to get the $\frac{dy}{dx}$ all alone so we divide by $2y$.

$$\frac{dy}{dx} = \frac{\arccos x}{2y}$$

8. Find y' if $3y^2 = 2x + x^2y$

Solution: In the above example we are going to take a straight derivative of each of the three terms $3y^2$, $2x$ and x^2y. What we do is then try to isolate y' (what equals the derivative) so in this manner we can have a derivative for the equation at hand. When doing implicit differentiation the derivative of y is y'. We have to note that the derivative of x is taken in the normal way of a variable, so that the derivative of x is 1 (for example). The reason for this is that y is a function of x, and does not exist without being dependent on the equation on the other side. This implies that x exists all along the domain.

The next thing to note is that the derivative of y^2 is $2yy'$. The reason is that you actually use the chain rule to solve x^2, which becomes $2x * 1$ hence $2x$. This result is obtained because the derivative of the inside function x, is one. Comparatively the derivative of the inside function, y is y'. So the outside function becomes $2y$, and the inside becomes y', we can multiply them together and we have $2yy'$. So the derivative of $3y^2$ is $6yy'$. The whole problem is shown as follows;

$$\begin{aligned} 3y^2 &= 2x + x^2y \\ 6yy' &= 2 + 2xy + x^2y' \end{aligned}$$

The method and explanation is as follows:

a) Differentiate both sides.

$$6yy' = 2 + 2xy + x^2y'$$

b) Now send all terms with y' inside to one side of the equation, and all terms without to the other side of the equal sign.

$$6yy' - x^2y' = 2 + 2xy$$

c) So on the side of y' terms, factor out the y'.

$$y'(6y - x^2) = 2 + 2xy$$

d) We can now safely divide both sides of the equation by the parentheses factor on the y' side.

$$y' = \frac{2 + 2xy}{6y - x2}$$

This is the final answer and we can use the same steps for other implicit functions.

9. Find the equation tangent to the circle $x^2 + y^2 = 36$ at the point $(4, 5)$.

Solution: Differentiate both sides of the equation:

$$\begin{aligned} \frac{d}{dx}(x^2 + y^2) &= \frac{d}{dx}(36) \\ \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) &= \frac{d}{dx}(36) \\ 2x + 2y\frac{dy}{dx} &= 0 \text{ (Chain Rule)} \\ \frac{dy}{dx} &= -\frac{x}{y} \end{aligned}$$

At the point $(4, 5)$, $x = 4$ and $y = 5$ so

$$\frac{dy}{dx} = -\frac{4}{5}$$

Therefore

$$y - 5 = -\frac{4}{5}(x - 4) \text{ OR } 4x + 5y = 41$$

10. Find $\dfrac{dy}{dx}$ of $x^2y + xy^2 = 3x$ by implicit differentiation.

Solution:

Step 1: You must differentiate both sides of $x^2y+xy^2 = 3x$ with respect to x, regarding y as a function of x. Remember that the product rule applies to the terms x^2y and xy^2. So,

$$(x^2)\frac{dy}{dx} + (2x)(y) + (x)\left(2y\frac{dy}{dx}\right) + (1)(y^2) = 3$$

Step 2: Combine the terms so you get:

$$x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2 = 3$$

Step 3: Because we are trying to solve the equation for $\dfrac{dy}{dx}$, move and isolate this term to the same side.

$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = 3 - y^2 - 2xy$$

Step 4: Divide the right side of the equation, $3 - y^2 - 2xy$, by $x^2 + 2xy$ to isolate $\dfrac{dy}{dx}$ on the left side of the argument. You will end up with:

$$\frac{dy}{dx} = \frac{3 - y^2 - 2xy}{x^2 + 2xy}$$

Because there is no further simplification possible for this problem, that is your answer.

11. Find y' if $x^4 + y^5 + 7 = x^2 + y^2 + y$

Solution:

$$\begin{aligned}
\frac{d}{dx}(x^4 + y^5 + 7) &= \frac{d}{dx}(x^2 + y^2 + y) \\
4x^3 + 5y^4y' + 0 &= 2x + 2yy' + y' \\
4x^3 - 2x &= 2yy' + y' - 5y^4y' \\
4x^3 - 2x &= (2y + 1 - 5y^4)y' \\
\frac{4x^3 - 2x}{2y + 1 - 5y^4} &= y'
\end{aligned}$$

Derivatives of log functions

12. Find $f'(x)$ if $f(x) = \dfrac{x}{\ln x}$.

Solution:

$$\begin{aligned} f'(x) &= \frac{(1)\ln x - x\left(\frac{1}{x}\right)}{(\ln x^2)} \quad \text{use the quotient rule} \\ f'(x) &= \frac{\ln x - 1}{(\ln x)^2} \end{aligned}$$

13. Find y' if $y = \log_7 \left(\dfrac{7}{6x-3}\right)$.

Solution:

$$y' = \frac{1}{\ln 7 \left(\dfrac{7}{6x-3}\right)} \frac{d}{dx}\left(\frac{7}{6x-3}\right)$$

-The derivative of this log base 7 is equal to one divided by natural log 7. You must bring along the argument with the natural log to the denominator and multiply this whole expression times the derivative of the argument.

$$y' = \frac{1}{\dfrac{7\ln 7}{6x-3}} \left(\frac{(6x-3)(0)-(7)(6)}{(6x-3)^2}\right)$$

-By using the quotient rule, the derivative of the argument transforms into this differentiated expression.

$$\begin{aligned} y' &= \frac{1}{\dfrac{7\ln 7}{6x-3}} \left(\frac{-42}{(6x-3)^2}\right) \\ &= \frac{6x-3}{7\ln 7}\left(\frac{-42}{(6x-3)^2}\right) \\ &= \frac{-6}{\ln 7(6x-3)} \end{aligned}$$

-Simplify

14. Find $f'(x)$ if $f(x) = \log_{10}\left(\dfrac{x}{x-1}\right)$.

Solution:

$$\begin{aligned} f'(x) &= \frac{1}{\left(\dfrac{x}{x-2}\right)\ln 10}\frac{d}{dx}\left(\frac{x}{x-2}\right) \\ &= \frac{1}{\dfrac{x\ln 10}{x-2}}\frac{(1)(x-2)-(x)(1)}{(x-2)^2} \\ &= \frac{x-2}{x\ln 10}\cdot\frac{x-2-x}{(x-2)^2} \\ &= -\frac{2}{x(x-2)\ln 10} \end{aligned}$$

Logarithmic Differentiation

15. Differentiate the function $y = \dfrac{x^3}{(1-10x)\sqrt{x^2+2}}$

Solution: Differentiating this function could be done with a product rule and a quotient rule. However, that would be a fairly messy process. We can simplify things somewhat by taking logarithms of both sides.

$$\ln y = \ln\left(\frac{x^3}{(1-10x)\sqrt{x^2+2}}\right)$$

Of course, this isn't really simpler. What we need to do is use the properties of logarithms to expand the right side as follows.

$$\begin{aligned} \ln y &= \ln(x^5) - \ln\left((1-10x)\sqrt{x^2+2}\right) \\ \ln y &= \ln(x^5) - \ln(1-10x) - \ln\left(\sqrt{x^2+2}\right) \end{aligned}$$

Notice when you take the logarithm of both sides the denominator comes up and is subtracted from the numerator with the ln being distributed throughout the problem. This doesn't look all the simple. However, the differentiation process will be simpler.

What we need to do at this point is differentiate both sides with respect to x. Note that this is really implicit differentiation.

$$\begin{aligned} \frac{y'}{y} &= \frac{5x^4}{x^5} - \frac{-10}{(1-10x)} - \frac{\frac{1}{2}(x^2+2)^{-1/2}(2x)}{(x^2+2)^{1/2}} \\ \frac{y'}{y} &= \frac{5}{x} + \frac{10}{(1-10x)} - \frac{x}{x^2+2} \end{aligned}$$

To finish the problem all that we need to do is multiply both sides by y and the plug in for y since we do know what that is.

$$\begin{aligned} y' &= y\left(\frac{5x^4}{x^5} - \frac{-10}{(1-10x)} - \frac{\frac{1}{2}(x^2+2)^{-1/2}(2x)}{(x^2+2)^{1/2}}\right) \\ &= \frac{x^3}{(1-10x)\sqrt{x^2+2}}\left(\frac{5}{x} + \frac{10}{(1-10x)} - \frac{x}{x^2+2}\right) \end{aligned}$$

That was probably a much simpler process of finding the derivative than simply using the quotient and product rule. The answer is probably a lot simpler as well.

Higher Derivatives

16. Find the fiftieth derivative of $F(x) = \dfrac{1}{x}$

Solution: We could find it by calculating each derivative separately until we finally got to the fiftieth derivative. However, there is an easier way to go about doing this. We would first calculate the first few derivatives and see if we could find a pattern.

$$\begin{aligned} F(x) &= 1/x \\ &= (1)x^{-1} \\ F'(x) &= -1x^{-2} \\ F''(x) &= +1 \cdot 2x^{-3} \\ F'''(x) &= -1 \cdot 2 \cdot 3x^{-4} \end{aligned}$$

As you can see from the equations above, we have a few patterns. First, the power of the variable decreases by one for each derivative, we could represent this by using $-n-1$ for the power of the variable as the power is the negative of the n^{th} derivative minus one. Next, the sign of the derivative changes back and forth from positive to negative, we could use $(-1)^n$, because, when n is odd, the n^{th} derivative is negative

and when n is even, the n^{th} derivative is positive. Finally, we can see that the integers being multiplied together are increasing by one, so we can represent this by $1 \cdot 2 \cdot 3 \cdots n$.
When put all together the new equation to find the n^{th} derivative will be,

$$F^{(n)} = (-1)^n \cdot 1 \cdot 2 \cdot 3 \cdots n x^{-n-1} = (-1)^n n! x^{-n-1}$$

17. Find $f''(x)$ if $f(x) = x^3 \tan x$.

Solution: The first step would be to find the first derivative of $f(x)$, because this will then help to achieve the second derivative.

1.) Begin with the derivative of $f(x) = x^3 \tan x$ first by utilizing the Product rule:

$$f'(x) = x^3 \frac{d}{dx}(\tan x) + \frac{d}{dx}(x^3) \tan x$$

This is then equal to:

$$f'(x) = x^3 \sec^2 x + 3x^2 \tan x$$

Being able to get the first derivative is the most crucial step for getting the correct second derivative. In order to get the second derivative one needs to then move on with the first derivative and take the derivative of this function one more time. This is a step where many people get confused, but if attacked properly it is very systematical.

2.) Begin with the first derivative, $f'(x) = x^3 \sec^2 x + 3x^2 \tan x$, and take the derivatives of all these steps to get the second derivative. In this step the Product rule will need to be utilized again on both sets of numbers in order to get the correct end product.

$$f''(x) = x^3 \frac{d}{dx}(\sec^2 x) + \frac{d}{dx}(x^3)(\sec^2 x) + 3x^2 \frac{d}{dx}(\tan x) + \frac{d}{dx}(3x^2) \tan x$$

This is then equal to:

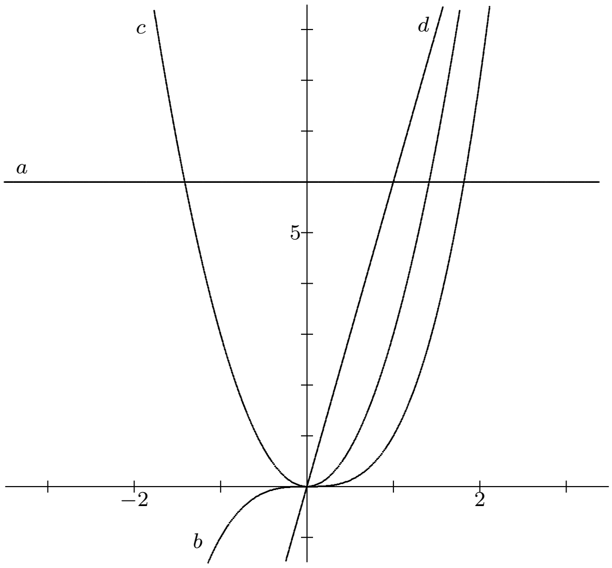
$$f''(x) = x^3[2(\sec x)(\sec x \tan x)] + 3x^2(\sec^2 x) + 3x^2(\sec^2 x) + 6x(\tan x)$$

This can then be simplified by combing the like terms of $\sec^2 x$:

$$f''(x) = 2x^3 \sec^2 x \tan x + 6x^2 \sec^2 x + 6x \tan x$$

The second derivative is now completed by first successfully getting the first derivative. Higher derivatives are a simple game of checking ones steps to make sure they are congruent with what needs to be found.

18. The figure shows graphs of f, f', f'', and f'''. Identify each curve and explain your choices.



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Solution: One can see the horizontal line (a) has to be the third derivative because it's derivative would be $f(x) = 0$ and $y = 0$ is not pictured. Therefore, from there you can see that the horizontal line is the derivative of the other line (d) because this line would just be a number (horizontal line) if the derivative was taken of it. Next, one can see that the parabola (b) is what the line $f''(x)$ is a derivative of because it flattens where the second derivative crosses zero. It also has a positive slope that matches the way the $f'(x)$ moves. From here one can infer by elimination that the actual function is the function that looks like an x^3 function (c). So we have

$$\begin{aligned} f(x) &\ - \ c \\ f'(x) &\ - \ b \\ f''(x) &\ - \ d \\ f'''(x) &\ - \ a \end{aligned}$$

19. If $f(x) = \dfrac{1}{\sqrt{x}}$, find a formula for the $n$the derivative $f^{(n)}(x)$.

Solution: The easiest way to finding the nth derivative of a function is to find parts of the derivative then assemble all the parts to solve for the nth derivative. First find $f^{(4)}(x)$ and once this is found a pattern can be noted, which will provide one piece to the nth derivative. It is also useful in understanding the pattern if once taking the derivative, the values in front of x are not multiplied.

$$\begin{aligned}
f(x) &= x^{-1/2} \\
f'(x) &= \left(-\frac{1}{2}\right)\mathbf{x}^{-\mathbf{3}/\mathbf{2}} && \text{odd derivative, } -\text{sign} \\
f''(x) &= \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\mathbf{x}^{-\mathbf{5}/\mathbf{2}} && \text{even derivative, } +\text{sign} \\
f'''(x) &= \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\mathbf{x}^{-\mathbf{7}/\mathbf{2}} && \text{odd derivative, } -\text{sign} \\
f^{(4)}(x) &= \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)\mathbf{x}^{-\mathbf{9}/\mathbf{2}} && \text{even derivative, } +\text{sign}
\end{aligned}$$

From this example a definite pattern can be seen. The pattern seen of

$$\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)$$

can be denoted as

$$\frac{1\cdot 3\cdot 5\cdot 7\cdots(2n-1)}{2^n}$$

Since we know that 2 multiplied by any number will be even, then 2 multiplied by any number minus one will always be odd. Since we want all odd values, the expression $2n-1$ is used. This is divided by two raised to the nth power, because for three derivatives the denominator will be multiplied by three times, and since the denominator never changes, this can be denoted as 2^n . Thus this first part of the equation works at defining the first part of the equation (red) for any derivative, n. From this pattern it can also be noted that there is a sign change with every successive derivative as seen in blue. In order to compensate for the pattern of negative, positive, negative, a $(-1)^n$ is placed before our equation. Because $(-1)^n$ is negative when n is odd and positive when n is even, the expression is placed at the beginning of the derivative to compensate for the sign change that occurs between successive derivatives. With the incorporation of this new information our equation looks like this

$$(-1)^n\frac{1\cdot 3\cdot 5\cdot 7\cdots(2n-1)}{2^n}$$

If the pattern of each successive derivative was positive, negative, positive then $(-1)^{(n-1)}$ would be placed in front of the equation. Next we are going to multiply our current equation by x raised to a certain power. The power x is raised to has to do with multiplying by x in our derivative, which was done at the end of each derivative and is highlighted in bold. Since we know that the numerator of what we are raising x to is odd, then we use $2n-1$ to express this. The denominator of what we are raising x to will always be 2, because we can see that this value stays constant with each successive derivative (green). A negative sign is placed before this expression, due to our original function being raised to a negative power. When we put all of the components together we get an equation that looks like:

$$f^{(n)}(x) = (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2^n} x^{-(2n-1)/2}$$

With this equation we can now find any derivative of the function $f(x) = \frac{1}{\sqrt{x}}$ by inserting whatever number derivative we need to find in for n.

20. Calculate y' and y'' if $y = \ln|\sec 5x + \tan 5x|$

Solution:

$$\begin{aligned}
y' &= \frac{1}{\sec 5x + \tan 5x}(5\sec 5x \tan 5x + 5\sec^2 5x) \\
&= \frac{5\sec 5x(\tan 5x + \sec 5x)}{\sec 5x + \tan 5x} \\
&= 5\sec 5x \\
y'' &= 5(\sec 5x \tan 5x)(5) \\
&= 25(\sec 5x \tan 5x)
\end{aligned}$$

21. Find a formula for $F'''(x)$ if $F(x) = \frac{x}{(2x-1)^2}$.

Solution:

$$\begin{aligned}
F(x) &= x(2x-1)^{-2} \\
F'(x) &= (x)\frac{d}{dx}((2x-1)^{-2}) + (2x-1)^{-2}\frac{d}{dx}(x) \\
F'(x) &= x(-2(2x-1)^{-3}(2)) + (2x-1)^{-2}(1) \\
F'(x) &= -4x(2x-1)^{-3} + (2x-1)^{-2} \\
F''(x) &= (-4x)\frac{d}{dx}[(2x-1)^{-3}] + (2x-1)^{-3}\frac{d}{dx}(-4x) - 2(2x-1)^{-3}(2) \\
F''(x) &= -4x(-3(2x-1)^{-4}(2)) + (2x-1)^{-3}(-4) - 4(2x-1)^{-3} \\
F''(x) &= 24x(2x-1)^{-4} - 8(2x-1)^{-3} \\
F'''(x) &= [(24x)\frac{d}{dx}(2x-1)^{-4} + (2x-1)^{-4}\frac{d}{dx}(24x)] - [(8)\frac{d}{dx}(2x-1)^{-3} + (2x-1)^{-3}\frac{d}{dx}(8)] \\
F'''(x) &= [24x(-4(2x-1)^{-5}(2)) + (2x-1)^{-4}(24)] - [8(-3(2x-1)^{-4}) + (2x-1)^{-3}(0)] \\
F'''(x) &= [-192(2x-1)^{-5} + 24(2x-1)^{-4}] + [24(2x-1)^{-4} + 0] \\
F'''(x) &= -\frac{192}{(2x-1)^5} + \frac{48}{(2x-1)^4}
\end{aligned}$$