

Reading Guide for Morris Kline's *Mathematics in Western Culture*

I. Introduction

1. What aspects of mathematics are missing if we consider the subject as merely a series of techniques?
2. What is meant by “postulational thinking”?
3. What are some driving forces that motivate people to study mathematics? List 3 – 5 that come to mind.
4. In what ways is mathematics considered beautiful?
5. In mathematics, what does one mean by “truth”?
6. Comment on the use of mathematics to “control” or “have power” over nature.

II. The Rule of Thumb in Mathematics

7. What is place value? List modern remnants of number bases other than base 10.
8. In reading Chapter II, consider the following groups of people and list contributions they made to mathematics:

Egyptians Babylonians Greeks Hindus
9. What motivated each of the four groups above to contribute what it did to mathematics?
10. How do we show today that, as stated on page 23, “of all rectangles with a given area, the square has the least perimeter”?
11. In what ways has the mathematics of these people been involved with their religion?

III. The Birth of the Mathematical Spirit

12. In what major ways did Greek mathematics differ from that of the Egyptians and Babylonians?
13. What is deductive reasoning?

14. Why did the Greeks prefer abstract concepts?
15. Note Plato's worldview.
16. Why did the Greeks study geometry? Compare this to the motivation for the Egyptians and Babylonians.
17. Note some examples of ways in which the Greeks expressed arithmetic ideas geometrically.

IV. The *Elements* of Euclid

18. Who were the Pythagoreans, and what contributions did they make?
19. Describe the structure of Euclid's *Elements*.
20. Of what use were the five types of regular polyhedra?
21. List the three famous mathematical problems of the Greeks. How were they to be proven? When were they proven?
22. Prove the theorem associated with *Figure 9*, page 52.
23. What is an "art", and why did the Greeks view mathematics as one? Do you concur?
24. Discuss the Greek attitude toward the infinite. How is this evident in their temples and art?
25. Summarize the ancient Greek contributions that determine the character of modern mathematics.

V. Placing a Yardstick to the Stars

26. Characterize Alexandria, Egypt as an intellectual center; during what period of time was it in its glory? What factors contributed to its greatness?
27. List six great Alexandrian mathematicians, including Archimedes; for what are they known?
28. Was Columbus (1492) the first to believe the world was round? Explain.
29. Describe how Hipparchus found the distance to the moon.

VI. Nature Acquires Reason

30. How did the Greek approach to the study of nature differ from previous civilizations?
31. Give some examples which serve to reinforce the Pythagorean idea that number was necessary in the explanation of the universe. In your discussion include geometry, music, astronomy, and human relationships.
32. In studying the motion of the planets, how did the Greek approach differ from the Egyptian?
33. Describe how Euclid's geometry gave evidence that nature had a rational plan.
34. Give several examples of the mathematical structure of the universe.
35. Explain Hipparchus' theory of the motion of planets. How does this compare with the heliocentric theory?
36. Why was it hard for Christians to give up the idea that the earth is the center of the universe?
37. Compare and contrast Greek and Roman attitudes toward abstraction and application.
38. In what ways were the Romans to change the whole intellectual world?
39. Who was Hypatia?

VII. Interlude

40. Describe the use of the Bible as a scientific document. What about its use today?
41. What were early Christian views toward learning?
42. How did the Church fill in when the Roman empire fell?
43. Who introduced negative numbers, and when? What other contribution(s) did they make? Why do you suppose these developments happened when and where they did?
44. What was the relationship between physicians, mathematicians, and astrologers during this period?
45. Describe how the Church's approach to reason and Aristotle change. What is the situation today?

46. With what belief did late medieval scholars approach the study of nature?

VIII. Renewal of the Mathematical Spirit

47. Who was Jerome Cardan (Gerolamo Cardano)?

48. Describe the relative importance of the ancient Greeks to the Renaissance intellectuals.

49. Characterize the commercial changes evident at the time of the Renaissance and some of their implications.

50. Give several influences which stimulated the advancement of mathematics at this time. Include scientific advancement and the Church.

51. What would have been meant when a scientist of this time said that mathematics was the key to nature's behavior?

52. Discuss Descartes' concept of the role of mathematics in describing space and time.

53. How did people in the 1600s reconcile Christianity and science?

54. Discuss the statement, "The Renaissance scientist was a theologian with nature instead of God as his subject" (109). What is the driving motivation behind scientists and science students today?

55. Elaborate on the distinction between the mathematical and experimental approach to the study of nature.

IX. The Harmony of the World

56. Compare the Copernican and the Ptolemaic theories of the heavens.

57. Discuss several interesting but questionable applications of mathematics made by Johannes Kepler.

58. What were Kepler's three laws of planetary motion?

59. Give several early objections to the theories of Copernicus and Kepler; what were some of the Protestant reactions?

60. How did the heliocentric theory help to shape modern thought?
61. React to the few lines from Milton's *Paradise Lost* on page 122.

X. Painting and Perspective

62. What need did the Renaissance painters have for mathematics?
63. What are some of the roles assumed by the artist of this period? What do you see as the roles assumed by artists today?
64. Contrast Medieval and Renaissance art. What role does art play for you today?
65. Discuss the art reproduced as plates VII through XIII. Comment on the apparent trend in painting that is in evidence.
66. Identify some of the following: Duccio, Giotto, Lorenzetti, Brunelleschi, Leon Battista Alberti, Leonardo da Vinci, Albrecht Durer.
67. In what ways is it evident that Signorelli, Bramante, Michelangelo, and Raphael were especially concerned with mathematics in their work?
68. Give the essential theorems of perspective.
69. Of what value is the use of perspective generally? What were some of the shortcomings in Renaissance perspective?
70. Make drawings of the following, using perspective:
 - (a) A room with square tiles on the floor.
 - (b) A room with a rectangular table on the floor, parallel to the back of the room.
 - (c) A table at an angle to the back of the room.
71. Discuss plates XV through XXV.

XI. Science Born of Art

72. Contrast Euclidean geometry and the new projective geometry.
73. What are some questions artists raised for mathematicians? Where do mathematicians' questions come from today? What questions are artists raising today?
74. Identify and give the theorems of Desargues, Pascal, and Brianchon.
75. What are some mathematically significant problems associated with map making?
76. Compare and contrast stereographic projection, cylindrical projection, and Mercator projection.

XII. A Discourse on Method

77. Why did Descartes believe a study of mathematics was necessary for him in his search for truth?
78. What were the four principles which guided Descartes in his method of determining truth?
79. List the "building blocks" of Descartes' philosophy.
80. What reasoning led Descartes to conclude the existence of God? Does his reasoning resonate with you?
81. Compare the philosophy of Descartes with that of previous philosophers.
82. How did Pierre Fermat compare with Descartes?
83. Describe the development of mathematics up to the seventeenth century. What at this time motivated the expansion of this area?
84. In what ways were Descartes and Fermat dissatisfied with Euclidean geometry?
85. Discuss how the new geometry as introduced by Descartes is a combination of algebra and geometry.

XIII. The Quantitative Approach to Nature

86. Describe the situation which led Galileo to revolutionize the study of science. Compare his approach to former scientific studies.

87. Discuss the implications of describing natural phenomena mathematically.
88. What were some principles of Aristotle that Galileo had to counter?
89. Discuss the formulas describing motion of a freely falling body.
90. What are some lessons to learn from the experiences of Galileo? Speculate on why the Church did not admit error in regard to its dealings with Galileo until November 1992.

XIV. The Deduction of Universal Laws

91. Who was Isaac Newton? How did he view himself? What three giant steps did Newton make in science?
92. What were some publications of Newton, and how were they received? How did Newton's work complement that of Galileo and Kepler?
93. Why does the moon stay in orbit about the earth? Explain the formula $F = \frac{kMm}{r^2}$, then summarize Newton's contribution to the theory of gravity.
94. Can one develop Kepler's laws from Newton's? Do Newton's laws demonstrate the power of mathematics? What are some specific phenomena described by Newton's laws?
95. Investigate why Newton was interested in the discoveries being made at the pyramids.
96. Discuss the discovery of Neptune. Did the laws of gravitation explain all celestial mechanics, or were there remaining irregularities?
97. Who was Pierre Simon Laplace? Speculate on his worldview, and compare with some common worldviews found on campus.

XV. The Calculus

98. Who was Gottfried Wilhelm Leibniz? Do personal rivalries play a role in the development of mathematics?

99. Distinguish between average and instantaneous rates of change. Explain the statement that instantaneous speed “is the number approached by average speeds as the intervals of time over which the average speeds are computed approach zero.” (Kline, 220)
100. What does the formula $d = 16t^2$ explain? What are some applications of the concept of “rate of change”?
101. Compare/Contrast the Principle of Least Time and the Principle of Least Action.
102. What is your general reaction to these chapters? What role does mathematics play in describing the universe?

XVI. Newtonian Influence: Science and Philosophy

103. Describe Kline’s main argument for a discernible Newtonian influence on science and philosophy. Are you convinced? What do you see as Newtonian in science and philosophy? How has this influence affected our culture? Include any of your personal thoughts or reactions.

XVII. Newtonian Influence: Religion

104. Has religion been impacted by Newtonian developments? If yes, how? If not, why? Comment on Kline’s view of the relationship between Newtonian influences and religion. Include any of your personal thoughts or reactions.

XVIII. Newtonian Influence: Literature and Aesthetics

105. How did literature respond to Newtonian influences? Were these influences sustainable to the present century? Include any of your personal thoughts or reactions.
106. Of the three chapters (XVI,XVII,XVIII), which seems to illustrate the clearest connection between Newtonian theory and the given discipline? Explain.

XIX. The Sine of G Major

107. Note the implications of Fourier's Theorem, including its relation to the telephone.

XX. Mastery of the Ether Waves

108. Discuss the main contributions of James Clerk Maxwell. Include an explanation of the switch of the foundations of physics, and Maxwell's relation to determinism.

XXI. The Science of Human Nature

109. Discuss the thought of John Locke; include references to the Declaration of Independence.

110. Discuss Jeremy Bentham's thought and Utilitarianism. Refer to Bentham's fundamental axiom and his simple ethic.

111. Respond to the following from the top of 332: "Perhaps [political theorists] did no more than justify and proclaim the political emergence of the common man."

112. Respond to the following from 332: "Equally axiomatic are the rights to liberty, property, and security, and the proposition that land and (or) labor are the sole sources of wealth."

113. Discuss the postulata of Thomas Malthus and the law of wages of David Ricardo.

XXII. The Mathematical Theory of Ignorance

114. Describe the *a priori*, deductive approach to solving problems. Why does it often fail to be useful?

115. What were the main objectives of Graunt and Petty?

116. Define three common averages used to summarize a collection of numbers. What do they fail to convey?

117. Briefly describe the properties of a *normal distribution*, or normal frequency curve.

118. Note Adolphe Quetelet's (1796-1874) attempt to discover characteristics of the "average man".

119. The statistician Francis Galton (1822-1911) wanted to employ the ideas of Quetelet to “mathematize Charles Darwin’s theory of evolution by looking at the inheritance of variation.”¹ What new idea did Galton contribute?
120. What are some dangers of drawing precise conclusions from statistical analyses?

XXIII. Prediction and Probability

121. Explain Pascal’s triangle and some of its uses.
122. Comment on the thought of Pascal, and Pascal’s wager.

XXV. The Paradoxes of the Infinite

123. Describe Georg Cantor’s contribution to the study of the infinite. How was Cantor’s work received?
124. Give examples of how the idea of *one-to-one correspondence* is used to deal with infinite sets.
125. Identify one of Zeno (490-430 BC) of Elea’s paradoxes and explain how Cantor’s theory might help resolve it.
126. Comment on one of the following paradoxes which involve classes of objects: the Cretans, the barber, heterological words.
127. What is meant mathematically by \aleph_0 ? Give some examples.

XXVI. New Geometries, New Worlds

128. What distinguishes a Euclidean geometry from a non-Euclidean one?

XXVII. The Theory of Relativity

129. What is relativity?
130. Explain the significance of the Michelson-Morley experiment.

¹Victor J. Katz, *A History of Mathematics: An Introduction*, 2nd edition (New York: Addison Wesley, 1998), page 759.