Reading Guide for Amir D. Aczel's *The Mystery of the Aleph*

$\aleph_0$ Halle

1. Explain in your own words your understanding of the equation $2^{\aleph_0} = \aleph_1$.

$\aleph_1$ Ancient Roots

2. Who was Zeno of Elea, and what are his paradoxes?

3. List some of the ideas of Pythagoras and the Pythagoreans.

4. What is the method of exhaustion, and who was Eudoxus?

$\aleph_2$ Kabbalah

5. Briefly summarize what Kabbalah is.

6. Define gematria.

7. What are the Sefirot?

$\aleph_3$ Galileo and Bolzano

8. Galileo, in a dialogue, has Salviati set up a one-to-one correspondence between two sets. What are those sets, and what is the significance?

9. Explain the vacancy policy at Hilbert's Hotel.

10. How did Bolzano extend Salviati's example?

$\aleph_4$ Berlin

11. Describe the Riemann sphere and why it is important.

$\aleph_5$ Squaring the Circle

12. How is it that the rational numbers have measure zero?

13. Distinguish between algebraic and transcendental numbers. How many are there of each type?

$\aleph_6$ The Student

14. Who were some of Georg Cantor's teachers?

$\aleph_7$ The Birth of Set Theory

15. Who was Richard Dedekind, and what is a Dedekind cut?

16. Elaborate on the process by which Giuseppe Peano defined the nonnegative integers.

$\aleph_8$ The First Circle

17. Restate Georg Cantor's diagonalization argument in your own words. What does it prove?

$\aleph_9$ "I see it, but I don't believe it."

18. How does the dimension of a mathematical entity determine the number of points it contains?

19. Can the points of an n-dimensional continuous space be put into a one-to-one correspondence with the points on the real line? Why or why not?

$\aleph_{10}$ Virulent Opposition

20. Summarize the antagonism evident between Georg Cantor's view of mathematics and that of Kronecker.

$\aleph_{11}$ The Transfinite Numbers

21. Define the transfinite number ω.

22. Simplify $\aleph_0 + \aleph_0$ and $\aleph_0 \times \aleph_0$, explaining your reasoning in your own words.

$\aleph_{12}$ The Continuum Hypothesis

23. How many elements are in the power set of $\{1, 2, 3, 4\}$? In the power set of $\{1, 2, 3, \cdots, n\}$?

24. What is signified by the equation $c = 2^{\aleph_0}$?

25. State the continuum hypothesis.

$\aleph_{13}$ Shakespeare and Mental Illness

26. Speculate on why Georg Cantor might have become interested in the authorship of Shakespeare's works.

$\aleph_{14}$ The Axiom of Choice

27. State the well-ordering principle.

28. How did Ernst Zermelo's axiom of choice relate to the well-ordering principle and the continuum hypothesis?

29. What is the controversy surrounding the axiom of choice?

$\aleph_{15}$ Russell's Paradox

30. State Russell's paradox in terms of set theory.

31. Explain in your own words the Banach-Tarski paradox.

$\aleph_{16}$ Marienbad

32. List some of the main results from Cantor's legacy.

$\aleph_{17}$ The Viennese Cafe

33. What is Zorn's Lemma, and how is it related to Cantor's work?

34. Give the essence of Kurt Gödel's incompleteness theorem.

$\aleph_{18}$ The Night of June 14-15, 1937

35. What did Gödel prove concerning the axiom of choice and the continuum hypothesis?

$\aleph_{19}$ Leibniz, Relativity, and the U.S. Constitution

36. Describe Kurt Gödel's relationship with Albert Einstein.

37. What assumptions did Gödel make to solve Einstein's gravitational field equations?

38. What logical flaw did Gödel 'discover' in the U.S. Constitution?

$\aleph_{20}$ Cohen's Proof and the Future of Set Theory

39. Highlight what Paul Cohen was able to prove in 1963, and its significance in the area of mathematics begun by Georg Cantor.

40. Form a conjecture about the existence of large cardinal numbers.

$\aleph_{21}$ The Infinite Brightness of the Chaluk

41. Alan Turing, the father of modern computing, also established a result related to infinity. What was it?

42. Comment on the quote from Georg Cantor: "The essence of mathematics lies in its freedom."