The Cossic Art

Writing Algebra with Symbols

February 5, 2007

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- Solve for x: $2x^2 = 5 + 3x$

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- The English picked this up, calling it "the Cossike Arte", meaning "The Art of Things"

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- Good notation can even seem to think for us

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- Speaks of "the" unknown - this notation did not represent more than one unknown

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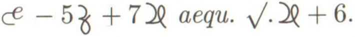
- In Christoff Rudolff's *Coss* of 1525 or Michael Stifel's *Arithmetica Integra* of 1544, the equation would be written: 

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- 1580s, Simon Steven of Belgium used circles around exponents. This work was carried to England in the early 17th century.

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- 1620s, Thomas Harriot: $5aaa + 7ee$
  1634, Pierre Herigone: $5a3 + 7e2$
  1636, James Hume: $5a^{iii} + 7e^{ii}$
  1637, Descartes: $5a^3 + 7e^2$

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- Notation itself then led to negative exponents, rational exponents, irrational exponents, and complex exponents